

# MATHEMATICAL MODELLING OF A TRANSFORMER WITH TAPS AT BOTH WINDING IN THE POSITIVE-SEQUENCE POWER FLOW

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**Abstract:** This paper addresses the issues related to transformer with load tap changer (LTC) facility on both winding to control voltage magnitude in power system network. The LTC correlated nodal admittances equations have been derived and added to the existing Newton-based power flow computer program for vastly robust iterative solution. The state variables are updated subsequently during each iteration and the associated tap's operation limit is also checked. The LTC modelling is carried out in MATLAB. The proposed LTC based model is implemented in IEEE-14 bus network to demonstrate the efficacy of the projected model and results are presented.

**Key Words:** Load taps Changers (LTC), Newton's method, Power Flow control, voltage magnitude control.

## 1. Introduction

Owing to the large industrial outputs, expedition for economic scale of operation, environmental demands have limited the extension of transmission network facilities which forced to explore new means of utilizing and maximizing the power transfer through the existing power system network. The transmission network has been considered in such ground and this movement may raise the issues related to the voltage collapse based on the power-voltage (P-V) relationship. Hence, instability of the system may occur following heavy loading in the transmission system [1-3]. Moreover, penetration of renewable energy also raises the voltage stability phenomenon in the existing electric grid [4-5].

Thus maintaining voltages within tolerable limit at all buses in electric system must be ensured in order to guarantee acceptable operation of the power system network including the reliable operation and protection of electrical appliances at customer end. Transformer is the fundamental electro-mechanical device to convert and provide the convenience of connecting two different voltage levels. Different voltage levels are controlled by adjusting the tap positions by physically shifting the turn's ratio of the transformer based on control signal i.e. load tap changer (LTC) to regulate network voltage at desired level [6-9]. Basic LTC arrangement of a transformer control is explained in [10]. Voltage control using LTC is a very traditional but yet cost-effective approach compared to additional Flexible AC transmission system (FACTS) devices to provide reactive power support in response to voltage regulation. To deal with voltage stability phenomena

arises from different operating phenomena, smart and stable LTC voltage control schemes is required.

In this paper, we have implemented load tap changer (LTC) mechanism to adjust the voltage operating limit where the control objective is constrained within maximum and minimum LTC taps and limits of the voltage magnitude in operating range. The model is developed with the idea that transformer taps are available in both primary and secondary windings. However, tap changing mechanism will be in operation on one side of the transformer at a given time while taps on other side will be fixed during that time.

## 2. Technical Background

### A. Basic Operation of LTC in a Transformer

Power systems operating at different voltage level need to be linked with Transformer. Transformation ratio is used to regulate voltage and is usually achieved by using taps on one of the windings. This way transformation ratio is changed stepwise rather continuous basis and this facility can be manual or automatic in control. In addition to changing voltage levels, transformers are also used to control voltage and are almost invariably equipped with taps on one or more windings to allow the turn's ratio to be changed [11]. Figure 1 shows the typical working principle of transformer tap changing mechanism. The transformer subjects to disturbances  $z(t)$  which is may be due to changes in network loading or network configuration. Figure 2 shows the typical symbol of transformer with taps facilities available on both sides. Selector switch is available in order to select the taps. Regulator receives the measurement of voltage  $V_T$  and current  $I_T$  on a chosen side of the transformer to provide control signal. Based on the error between the measure and the reference value, it creates a control signal and executes the control function to either step increase or decrease the taps ratio to regulate voltage.

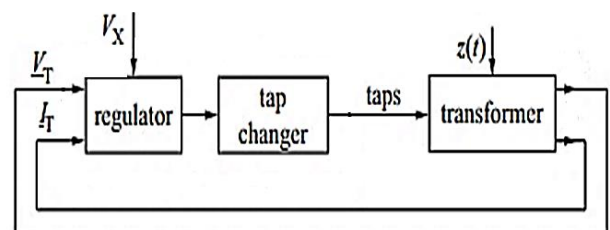


Fig. 1. Block diagram of typical transformation ratio control system

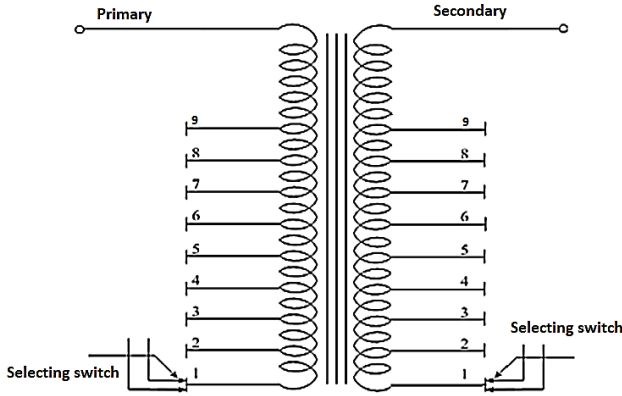


Fig. 2. Typical symbol of transformer with taps on primary and secondary

### B. Motivation behind present task

Power flow calculations are carried out in order to deal with system operation and control. These calculations involve solving set of non-linear algebraic equations representing the network under steady state condition.

Several approaches have been carried out for power flow solution in the past. Most of early iterative methods were based on the Gauss-Seidel Y-matrix which works fine with a small network. But as the size of network expands, number of iteration increases and thus slows down the procedures. Due to this slow converging characteristic, new iterative method became essential to be found. In large-scale power flow studies the Newton–Raphson method has proved most successful owing to its strong convergence characteristics [12-13].

$$[P] = f(V, \theta, G, B) \quad (1)$$

$$[Q] = g(V, \theta, G, B) \quad (2)$$

and their linearization around a base point,

$[(P^0, Q^0)]$  is,

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}^i = [J]^i \begin{bmatrix} \Delta \theta \\ \Delta V \end{bmatrix}^i \quad (3)$$

They are already well documented in [13].

Where the variables used in the above equations (1-3) are-

P and Q are vectors of real and reactive nodal power injections

V is the nodal voltage magnitudes and  $\theta$  is the angles

G is the network conductance and susceptance B.

$\Delta P$  is the active power mismatch vector

$\Delta Q$  is the reactive power mismatch vector

$\Delta \theta$  is the incremental change is nodal voltage angles

$\Delta V$  is the incremental change is nodal voltage magnitudes

J is the jacobian matrix

i is the iteration number

Following the reasons mentioned in [14], a comprehensive model of three winding single phase transformer with the provision of complex taps on

both primary and secondary winding are presented in [14]. However, the LTC based model doesn't require complex taps [15]. In this paper, mathematical modeling of the LTC based two winding transformer is made by making simplifying assumption in the single phase three winding transformer.

Another reason of having this model is the growing popularity of distributed generation (DG) penetration in the system. Traditionally the power flow direction is vertical but with the DG penetration, power flow direction in existing electric network changes when DG output power exceeds the local power demand [16-19]. Therefore excess power creates voltage stability concern in existing electric grid. So controlling the taps on both side of the transformer is quite demanding for the electric system. This makes the typical OLTC controlled mechanism quite inappropriate for the present and/or future intelligent power system network and became motivating area of development in control technique.

The aim of this paper work is to-

a) Modeling LTC with taps on the primary and secondary windings in the positive-sequence power flow

b) Make the transformers connected between nodes 4-7, 4-9 and 5-6 in the IEEE-14 bus test system as shown in Figure-3, to be LTC so that they regulate the voltage magnitudes at 1.01, 1.01 and 1.05, respectively.

During the implementation of mathematical modeling, conditions for taps limit should be checked and appropriate actions should be taken based on the conditions of limit violation.

### 3. IEEE-14 bus Test Network

In this study, IEEE-14 bus test system [20] is used to demonstrate the proposed LTC control algorithm. The system comprises 5 generators of which 3 acts as synchronous compensator to make available reactive power provision. The network is selected due to the reason of simplicity and availability of transformer in the network compared to IEEE-5 bus system. The corresponding generator, load and transformer values are given in Appendix-A.

### 4. LTC Mathematical Modeling

#### A. Power equations

Voltage control using the transformer tap is quite widely used method in the power system network. Transformer taps control may be manual or automatic control. For the modelling purpose, let's assume that the Transformer is connected between two nodes k and m. Voltage and Current at the corresponding nodes are  $V_k, I_k$  (considered as primary) and  $V_p, I_p$  (considered as secondary). The impact of magnetizing branch is negligibly small in power flow solution which will be ignored in this project.

- Ⓒ GENERATORS
- Ⓒ SYNCHRONOUS CONDENSERS

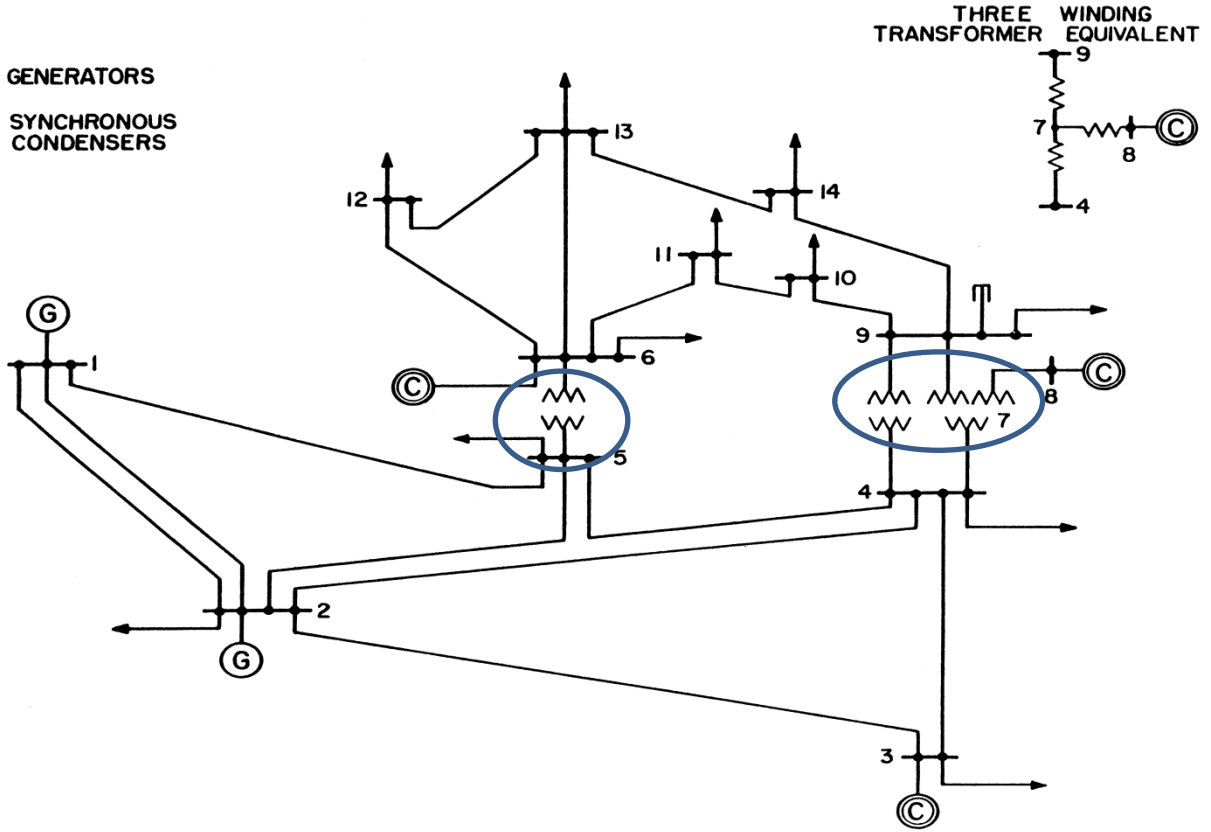


Fig. 3. IEEE 14-bus test system and corresponding LTC point [20]

Although, we have established a model which has a provision of tap changing mechanism in both side of the winding, but the tap on one side will only be active and varying while other side of the winding tap will remain constant at nominal value at that time.

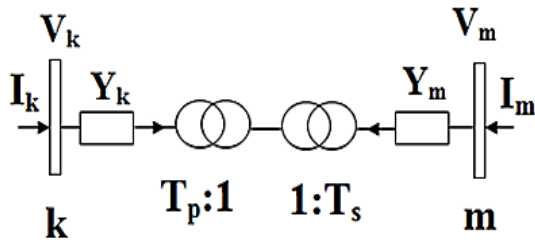


Fig. 4. Simple tap changing mechanism of transformer

The current and voltage existing in both windings are related by a matrix of admittance parameter as given by following expression:

$$\begin{pmatrix} I_k \\ I_m \end{pmatrix} = \begin{pmatrix} T_s^2 Y_{kk} & -T_s T_p Y_{km} Y_{mk} \\ -T_s T_p Y_{km} Y_{mk} & T_p^2 Y_{mm} \end{pmatrix} \begin{pmatrix} V_k \\ V_m \end{pmatrix} \quad (4)$$

The power flow equations at both ends of the transformer are derived where  $T_s$  and  $T_p$  are allowed to vary within design rating boundaries  $T_{p,\min} < T_p < T_{p,\max}$  and  $T_{s,\min} < T_s < T_{s,\max}$ .

$$P_k = V_k^2 T_s^2 G_{kk} + T_s T_p V_k V_m [G_{km} \cos(\theta_k - \theta_m) + B_{km} \sin(\theta_k - \theta_m)], \quad (5)$$

$$Q_k = -V_k^2 T_s^2 B_{kk} + T_s T_p V_k V_m [G_{km} \sin(\theta_k - \theta_m) - B_{km} \cos(\theta_k - \theta_m)], \quad (6)$$

$$P_m = T_p^2 V_m^2 G_{mm} + T_p V_m V_k [G_{mk} \cos(\theta_m - \theta_k) + B_{mk} \sin(\theta_m - \theta_k)] \quad (7)$$

$$Q_m = -T_p^2 V_m^2 B_{mm} + T_p T_m V_m V_k [G_{mk} \sin(\theta_m - \theta_k) - B_{mk} \cos(\theta_m - \theta_k)] \quad (8)$$

Where,

$$\begin{aligned} Y_{kk} &= G_{kk} + jB_{kk} = -Y_k \\ Y_{km} &= G_{km} + jB_{km} = -Y_k \\ Y_{mm} &= G_{mm} + jB_{mm} = -Y_m \\ Y_{mk} &= G_{mk} + jB_{mk} = -Y_m \end{aligned} \quad (9)$$

The set of linearized power flow equations for the nodal power injections, assuming the load tap changer (LTC) facility comprising at both side of the windings to regulate voltage magnitude at sending bus (k) and receiving end (m), may be written as:

$$\begin{bmatrix} \Delta P_k \\ \Delta P_m \\ \Delta Q_k \\ \Delta Q_m \end{bmatrix}^i = \begin{bmatrix} \frac{\partial P_k}{\partial \theta_k} & \frac{\partial P_k}{\partial \theta_m} & \frac{\partial P_k}{\partial T_p} T_p & \frac{\partial P_k}{\partial T_s} T_s \\ \frac{\partial P_m}{\partial \theta_k} & \frac{\partial P_m}{\partial \theta_m} & \frac{\partial P_m}{\partial T_p} T_p & \frac{\partial P_m}{\partial T_s} T_s \\ \frac{\partial Q_k}{\partial \theta_k} & \frac{\partial Q_k}{\partial \theta_m} & \frac{\partial Q_k}{\partial T_p} T_p & \frac{\partial Q_k}{\partial T_s} T_s \\ \frac{\partial Q_m}{\partial \theta_k} & \frac{\partial Q_m}{\partial \theta_m} & \frac{\partial Q_m}{\partial T_p} T_p & \frac{\partial Q_m}{\partial T_s} T_s \end{bmatrix}^i \begin{bmatrix} \Delta \theta_k \\ \Delta \theta_m \\ \frac{\Delta T_p}{T_p} \\ \frac{\Delta T_s}{T_s} \end{bmatrix}^i \quad (10)$$

The Jacobian elements in the LTC Jacobian matrix Equation based on the concept taps on both windings are given as follows:

$$\frac{\partial P_k}{\partial \theta_k} = -\frac{\partial P_k}{\partial \theta_m} = -Q_k - T_s^2 V_k^2 B_{kk} \quad (11)$$

$$\frac{\partial P_k}{\partial T_p} T_p = \frac{\partial P_k}{\partial T_s} T_s = P_k + T_s^2 V_k^2 G_{kk} \quad (12)$$

$$\frac{\partial Q_k}{\partial \theta_k} = -\frac{\partial Q_k}{\partial \theta_m} = P_k - T_s^2 V_k^2 G_{kk} \quad (13)$$

$$\frac{\partial Q_k}{\partial T_p} T_p = \frac{\partial Q_k}{\partial T_s} T_s = Q_k - T_s^2 V_k^2 B_{kk} \quad (14)$$

$$\frac{\partial P_m}{\partial \theta_m} = -\frac{\partial P_m}{\partial \theta_k} = -Q_m - T_p^2 V_m^2 B_{mm} \quad (15)$$

$$\frac{\partial P_m}{\partial T_s} T_s = \frac{\partial P_m}{\partial T_p} T_p = P_m + T_p^2 V_m^2 G_{mm} \quad (16)$$

$$\frac{\partial Q_m}{\partial \theta_m} = -\frac{\partial Q_m}{\partial \theta_k} = P_m - T_p^2 V_m^2 G_{mm} \quad (17)$$

$$\frac{\partial Q_m}{\partial T_s} T_s = \frac{\partial Q_m}{\partial T_p} T_p = Q_m - T_p^2 V_m^2 B_{mm} \quad (18)$$

Since we have mentioned earlier that only one tap changer mechanism will be effectively varying at a time, the linearized power flow equations will change at a following manner-

1. If load tap changer (LTC) is controlling nodal voltage magnitude at its sending end (bus k), 4<sup>th</sup> column in equation (10) will become PQ bus instead of PVT bus, which means- 4<sup>th</sup> column will be re-written as-

$$\frac{\partial P_k}{\partial V_m} V_m, \frac{\partial P_m}{\partial V_m} V_m, \frac{\partial Q_k}{\partial V_m} V_m, \frac{\partial Q_m}{\partial V_m} V_m$$

2. If nodal voltage magnitude control by the LTC takes place on its receiving end (bus m) as opposed to the sending end (bus k), the second and third columns in the LTC linearized equation (10) are interchanged, along with 4<sup>th</sup> column replaced by  $\frac{\partial P_k}{\partial V_m} V_m, \frac{\partial P_m}{\partial V_m} V_m, \frac{\partial Q_k}{\partial V_m} V_m, \frac{\partial Q_m}{\partial V_m} V_m$ , of equation (10) and new type Jacobian elements similar to the equations (11-18) are derived and used as entries in the linearized equation.

At the end of each iteration, i, tap controller value will be updated according to the following relation-

- If taps on primary (node k) is allowed to control-

$$[T_p]^i = [T_p]^{(i-1)} + \left[ \frac{\Delta T_p}{T_p} \right]^i [T_p]^{(i-1)} \quad (19)$$

- If taps on secondary (node m) is allowed to control-

$$[T_s]^i = [T_s]^{(i-1)} + \left[ \frac{\Delta T_s}{T_s} \right]^i [T_s]^{(i-1)} \quad (20)$$

Implementation of the tap changing mechanism required introduction of new type of bus named PVT bus. It comprises a generator PV bus but voltage is controlled by LTC rather than generator. In this bus nodal voltage magnitude and bus active and reactive power are specified, whereas LTC tap  $T_p$  or  $T_s$  (based on the tap controlling on node k or node m) act as state variable.

## B. Initializing state variable and limit checking

Normally, the initial value of the tap is selected at their nominal value. Hence,  $T_p = 1$  and  $T_s = 1$  for two winding LTCs are used. The status of LTCs is checked at the end of each iteration to check whether the LTCs are operating at their purpose, regulating voltage magnitude, while also operating within limits or not.

- a) LTC regulating nodal voltage magnitude at bus k with tapping facility at primary winding

$$T_{p,\min} < T_p < T_{p,\max} \quad (21)$$

If LTC tap  $T_p$  is operating with in the above mentioned limit, targeted voltage magnitude is achieved and the bus remains as PVT bus. However if the taps violates limit condition, it's no more remain a PVT bus. If any of the following condition takes place during the iteration process:

$$\begin{aligned} [T_p]^{(i+1)} + [\Delta T_p]^{(i)} &\geq T_{p,\max}, \\ [T_p]^{(i+1)} + [\Delta T_p]^{(i)} &\leq T_{p,\min}, \end{aligned} \quad (22)$$

bus k becomes PQ bus and tap is fixed at violated limit. The nodal voltage magnitude is allowed to vary at bus k and  $T_p$  is replaced by  $V_k$  as state variable and set of linearized equations will be changed accordingly.

Similarly the associated expressions can be explained for the tapping facility available in the secondary winding of the two winding transformer, mentioned below-

- b) LTC regulating nodal voltage magnitude at bus m with tapping facility at secondary winding

$$T_{s,\min} < T_s < T_{s,\max} \quad (23)$$

If LTC tap  $T_s$  is operating with in the above mentioned limit, targeted voltage magnitude is achieved and the bus remains as PVT bus. However if the taps violates limit condition, bus doesn't remain PVT bus anymore.

If any of the following condition takes place during the iteration process:

$$\begin{aligned} [T_s]^{(i+1)} + [\Delta T_s]^{(i)} &\geq T_{s,\max}, \\ [T_s]^{(i+1)} + [\Delta T_s]^{(i)} &\leq T_{s,\min}, \end{aligned} \quad (24)$$

bus m becomes PQ bus and tap is fixed at violated limit. The nodal voltage magnitude is allowed to vary at bus m and  $T_s$  is replaced by  $V_m$  as state variable and set of linearized equations will be changed accordingly.

## 5. Results And Discussion

LTC mathematical modeling based equations has been derived in order to analyze IEEE-14 bus testing network and carry out operation to achieve targeted voltage and analyze the results. There are three LTC based transformers in IEEE-14 bus network, connected between nodes 4-7, 4-9 and 5-6. Taps on one side of the transformer is allowed to vary at a given time and check the voltage magnitude and other associated constraints mentioned in the modeling.

### A. Results of LTC modeling in IEEE-14 bus network

Purpose of the LTC modeling is to varying transformer taps in order to achieve desired voltage magnitude at the node where transformer is connected. To carry out intended result, corresponding nodes are introduced as new bus, PVT bus, and solution has been carried out.

a) When regulating the voltage magnitude at node 4 & 5, the value of corresponding taps and voltage magnitude are given as follows-

Table 1  
Voltage Magnitude & Taps of 14-Nodes System at Sending end node

Voltage controlling node	Taps value		Voltage magnitude at corresponding node	Result
	TapS	TapP		
4	1.05	1	1.0101	Taps limit violated
4	1.05	1	1.0101	Taps limit violated
5	0.95	1	1.0160	Taps limit violated

Based on the results shown in Table-1, it can be concluded that at all nodes, taps limit condition is violated and therefore, the nodes become PQ bus and taps are fixed at their violated (maximum and minimum) limit. This justifies taps limit conditions as well changing the type of bus PVT into PQ type bus.

b) When regulating the voltage magnitude at node 7, 9 & 6, the value of corresponding taps and voltage magnitude has given in Table 2.

From the Table 2, it can be concluded that at node 9, desired voltage magnitude value is achieved while at node 7 & 6, taps limit violated and node 7 & 6 becomes PQ bus and taps is fixed at the violated limit.

Table 2  
Voltage Magnitude & Taps of 14-Nodes System at Receiving end node

Voltage controlling node	Taps value		Voltage magnitude at corresponding node	Result
	TapS	TapP		
7	1	1.05	1.0143	Taps limit violated
9	1	0.9908	1.01	Target is achieved
6	1	0.95	1.0183	Taps limit violated

### B. Discussion

Objective of this project is to attain targeted voltage magnitude at the nodes 4-7, 4-9 and 5-6 of IEEE-14 bus network by controlling LTC of the transformer operation. However, the main constraint is to maintain its operation within the operating limit. Moreover when the taps limit is violated, changing the corresponding bus as PQ bus. This ensures the maximum utilization of the transformer capacity to control the voltage at the Point of Common Coupling (PCC). If the transformer fails to control the desired voltage, additional arrangement to provide additional reactive power can be considered.

Based on the analysis of results obtained in both cases (at sending end and receiving end), we came into a conclusion that the purpose of the LTC modeling has been achieved.

## 6. Conclusion

The paper has introduced LTC based transformer operation with complex tap both in primary and secondary winding to regulate voltage magnitude at the connected node points and steady state analysis of LTC transformer was described. Initially, transformer taps act as a variable and voltage as fixed values. However, during the iteration process, variable can be changed based on conditions of the maximum and minimum taps position and hence, the actions are taken on changing conditions. Results show the effectiveness of the proposed modeling.

In future work, we will look into dynamic analysis to achieve more robust and accurate value of the taps position and voltage magnitude. Analyzing their behavior and response in a dynamic situation would be very demanding in the power system.

Moreover, the impact of distributed renewable energy sources penetration in the system will be considered as they also have growing impact on voltage in transmission and distribution system.

## Appendix-A

Table-1  
Values of generator and load at corresponding buses

Bus	Pgenerator(pu)	Load(pu)
1	2.3258- 0.3499i	0.2170 + 0.1270i
2	0.4000 + 0.4415i	0.9420 + 0.1900i
3	0.2349i	0.4780 - 0.0390i
4	0	0.0760 + 0.0160i
5	0	0.1120 + 0.0750i
6	0.1856i	0.2950 + 0.1660i
7	0	0.0900 + 0.0580i
8	0.2748i	0.0350 + 0.0180i
9	0	0.0610 + 0.0160i
10	0	0.1350 + 0.0580i
11	0	0.1490 + 0.0500i

Table-2  
Values of transmission line

Bus	r (pu)	x(pu)	B(pu)
1-2	0.01938	0.05917	0.0528
1-5	0.05403	0.22304	0.0492
2-3	0.04699	0.19797	0.0438
2-4	0.05811	0.17632	0.0374
2-5	0.05695	0.17388	0.0340
3-4	0.06701	0.17103	0.0346
4-5	0.01335	0.04211	0.0128
6-11	0.09498	0.19890	0
6-12	0.12291	0.25581	0
6-13	0.06615	0.013027	0
9-10	0.03181	0.08450	0
9-14	0.12711	0.27038	0
10-11	0.08205	0.19207	0
12-13	0.22092	0.19988	0
13-14	0.17093	0.34802	0

Table-1  
Values of initial voltages at buse

Bus	Voltage (pu)
1	1.06
2	1.045
3	1.010
4	1.0
5	1.0
6	1.060
7	1.0
8	1.060
9	1.0
10	1.0
11	1.0

Table-3  
Values of transformer

Bus	r(pu)	x(pu)
4-7	0	0.2091
4-9	0	0.55618
5-6	0	0.25202
7-8	0	0.17615
7-9	0	0.11001

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