

# An Adaptive Dynamic Implicitly Restarted Arnoldi method for the Small Signal Stability Eigen analysis

Y.CHABANE A.HELLAL

Laboratory of power system analysis and electrical network control, University of Amar Telidji  
Laghouat-ALGERIA, yasmina.gacem@gmail.com

**Abstract:** *This paper deals with the application of an adaptive Implicitly Restarted Arnoldi Algorithm based on Krylov subspaces coupled with a dynamic switching approach to the small signal stability eigen analysis problem for power systems. The goal of the modified algorithm is to converge quickly and directly to the sought critical eigenvalues concerned with poor damping ratios directly from the algorithm calculation.*

**Key words:** *Damping ratio, Dynamic switching, Eigenvalues, Implicitly Restarted Arnoldi Algorithm, Small Signal Stability*

## 1. Introduction.

Power system stability issue is considered as an important topic and had received a great deal of attention over years. We can classify power system stability into three categories as follows [1]:(i) rotor (power) angle stability, (ii) voltage stability and (iii) frequency stability.

The transient stability problem is one of the angular stability issues; it concerns the phenomenon which occurs the first few seconds following a short or a transient disturbance. As many efforts and interest were focused on this type of stability problem, well-established analytical techniques and computational programs are established to overcome and analyze the transient stability.

In the other side, ensuring sufficient damping of the system oscillations concerns the small signal stability problem. The system response to small changes is considered as an important requirement for the satisfactory operation of power systems. In this case of stability studies, the system can be linearized around its steady state operating conditions which is its equilibrium point in the same time. [2].

One of the most common methods for the Small Signal Stability Analysis SSSA is the eigenvalues calculation method. The method is well-known and has proved its efficiency [3]

Among linear techniques for solving SSSA problems, the QR-routine is so far the most used one. The corresponding algorithm calculates all the eigenvalues and eigenvectors of the system, and the operation needs time and memory storage for the spectrum values. For the SSSA we are mainly interested on few eigenvalues, those with a poor damping, and there is no need to consume time calculation and memory storage for the whole of the

spectrum, in this case, it's more interesting to consider the Krylov subspaces methods for finding the eigenvalues of interest. From these methods, the Arnoldi algorithm is the most efficient one; it provides good approximations for the few sought eigenvalues using orthogonal techniques [4].

Low frequency oscillations LFOs concern the rotor angle stability, the range of the oscillations is between 0.1-3Hz. The generators are more and more equipped with modern exciters to enhance the transient stability, the use of high-gain exciters, the HDVC converters or static Var compensators may create LFOs with poor damping.

LFOs include the oscillations resulting from the interaction between the mechanical and electrical modes of a generator-turbine system, as local modes: the most commonly encountered ones, control modes and torsional modes [5].

Inter-area modes describe the swinging of coherent groups of generators against another group, the range of frequencies is between 0.1-0.7 Hz. This type of oscillations is more complex than the local one and limits the quantity of power transmission on tie lines between the regions containing the groups of coherent generators. [6].

Power system stabilizers (PSSs) have been widely used as additional controllers to the exciters to damp out the LFOs, the PSS produces a component of torque in phase with rotor speed deviation to enhance the damping of the system and thus extends its capability transfer limits [7].

In this paper, we propose an algorithm adapted to SSSA stability which aims to find the few sought eigenvalues (those with the poorest damping ratios). The method is mainly based on an Adaptive Implicitly Restarted Arnoldi algorithm [8] coupled to a dynamic switching approach as proposed on [9] to accelerate the convergence to the wanted approximate eigenvalues.

The application of the adaptive dynamic algorithm to two test cases system has given satisfactory results and has shown the efficiency of the method in computing the wanted eigenvalues, especially when comparing the results with those calculated by the QR routine. Our method has taken less time for reaching the convergence with less storage memory space.

## 2. Small signal stability analysis.

For the SSSA issue, the power system can be described by a set of linear equations that are linearized around its equilibrium point represented by a steady state operating conditions [2].

### 2.1 State Space Model.

The equations modeling the state space model are borrowed from [10].

Complex dynamic equations of linear systems are generally represented by:

$$\frac{dX}{dt} = AX + BU \quad (1)$$

With:

$X$  is the system vector states,  $A$  is the state square matrix,  $B$  is the matrix which defines the proportion of each input applied to each state equation and  $U$  is the system vector inputs.

The system output is generally expressed as:

$$Y = CX + DU \quad (2)$$

With:

$Y$  is the system vector outputs,  $C$  is the output matrix and  $D$  is the feed forward matrix.

The solution of (1) is of the form:

$$X = \sum_{i=1}^n u_i z_i \quad (3)$$

$u_i$  is the  $i^{\text{th}}$  right eigenvector of  $A$ , and  $z_i$  the  $i^{\text{th}}$  mode that satisfies the following equation:

$$\frac{dz_i}{dt} = \lambda_i z_i + v_i BU \quad (4)$$

$\lambda_i, v_i$  respectively  $i^{\text{th}}$  eigenvalue and left eigenvector of  $A$

### 2.2 Eigenvalues and Stability Analysis

The eigenvalues  $\lambda_i$  for the matrix  $A$  are calculating by solving equation (5):

$$\det(A - \lambda I) = 0 \quad (5)$$

The  $i^{\text{th}}$  right eigenvector  $u_i$  satisfies:

$$Au_i = \lambda_i u_i \quad (6)$$

And the  $i^{\text{th}}$  left eigenvector  $v_i$  satisfies:

$$v_i A = \lambda_i v_i \quad (7)$$

The eigenvalues provide important feedback regarding the stability of the studied system, for example, the eigenvalues with real positive part corresponds to an unstable mode as each complex eigenvalue having the form  $(\sigma \pm j\omega)$  corresponds to an oscillatory mode.

Hence, an eigenvalue with a positive  $\sigma$  defines an unstable oscillatory mode, whereas, a pair with a negative  $\sigma$  represents a stable oscillatory mode.

The dominant modes are those associated to

eigenvalues with poorly damped oscillatory modes. The damping ratio  $\zeta$  defined in (8) provides a major indication to the SSSA. The higher damping ratio, the better damping effects to enhance the stability of the low frequency oscillations.

$$\xi = - \frac{\sigma}{\sqrt{\sigma^2 + \omega^2}} \quad (8)$$

## 3. The adaptive dynamic implicitly restarted Arnoldi algorithm.

The QR routine is a technique which calculates the whole of the spectrum, but for the SSSA issue, we mainly focus on analyzing the oscillatory modes, so only few eigenpairs from the spectrum are of interest. For this, the Krylov Subspace methods [11].are exploited.

Two widely used Krylov processes are: the Arnoldi process and the asymmetric Lanczos process [12].

As the state space matrix of power systems is generally a sparse matrix  $A$ . If we multiply  $A$  by  $x$  to get  $Ax$ , we can then multiply  $A$  by that vector to get  $Ax^2$ , and so we can build a Krylov subspace  $x, Ax, Ax^2, \dots$

The Arnoldi process is by far the most widely used Krylov process. The Arnoldi process begins with  $u_1 = cx$ ; where  $c = 1/\|x\|$

### 3.1 The Modified Implicitly Restarted Arnoldi Method (MIRA)

The implicit restart method, based on Sorensen's implicitly restarted Arnoldi process [8], is the implicitly shifted QR algorithm coupled to a k-step Arnoldi factorization.

The original algorithm for our modified Implicitly Restarted Algorithm was extracted from [13], the idea is to adapt it to our SSSA problem which is finding the eigenvalues of the critical oscillatory modes. For this, we introduce the damping ratio as a selective parameter; Instead of searching the eigenvalues with positive or largest real part as the ARPACK does very well [13].

We modify the basic Algorithm by adding one iteration(2.b) which calculate the damping ratios. When sorting the ratios, we obtain the wanted and unwanted eigenvalues. In such way we will only keep those affecting the small signal stability of the system.

Algorithm1: The Modified Implicitly Restarted Arnoldi Algorithm (MIRA)

1/ m-step Arnoldi Factorization

$$AV_m = V_m H_m + f_m e_m^T$$

- 2/ from  $\omega=1, \dots$ , until convergence
- 2.a/ Calculate the spectrum of  $H_m$ ,
  - 2.b/ calculate  $\zeta$  as in(8) and split the eigenvalues in two sets regarding their damping ratio: the  $k$  wanted ones and the  $p$  shifts ones
- $$\lambda(H_m^\omega) = \{\lambda_1^\omega, \dots, \lambda_m^\omega\} \cup \{\lambda_{k+1}^\omega, \dots, \lambda_m^\omega\}$$
- 2.c/ Carry out  $p$ -QR-implicitly shifted
  - 2.d/ Update  $V, H$  and  $f$
  - 2.e/ Apply  $k$ -step Arnoldi factorization
  - 2.f/ Extend the  $k$ -step Arnoldi factorization to length  $k+p=m$

The convergence is obtained when the maximum relative residual norm (res) falls below the defined tolerance [13]

$$res = \frac{\max \left( \left\| A - \lambda_i^\omega I x_i^\omega \right\| \right)}{\|A\|}$$

### 3.2 Adaptive Dynamic Implicitly Restarted Arnoldi Method (ADIRA)

For accelerating the convergence of the previous algorithm, a dynamic technique is used. The method was first proposed on [9], the idea is to exploit the relationship between the residual of the current step and the residual of the previous step to modify subspace dimension

The aim of our work is to accelerate the convergence of our Modified Implicitly Restarted Arnoldi algorithm by incorporating Dynamic Switching routine as proposed on [14]

Algorithm2: Adaptive Dynamic Implicitly Restarted Arnoldi method (ADIRA)

- 1/ Run an Arnoldi factorization of length  $k$ .
- 2/ Compute the approximate eigenpair  $(\lambda_1^0, y_1^0)$  and find the residual vector  $r_1^0$  and set  $\hat{\lambda}_1^0 = \lambda_1^0$
- 3/ Main Loop until convergence
  - a/ if  $\omega=1$ , set  $\Xi_1=1$ ; else compute  $\eta_1^\omega$  and  $\Xi_1$  if  $\lambda_1^{\omega-1}$  has not converged.
    - b/ Use the algorithm in Appendix A for switching the Krylov subspace dimension.
    - c/ Extend the length to  $m$  for Arnoldi decomposition.
    - d/ Same as steps (1) and (2) from algorithm 2 to obtain  $(\lambda_i^\omega, y_i^\omega)$ .
    - e/ if  $\lambda_1^\omega$  has not converged, the if  $\zeta(\lambda_1^\omega) > \zeta(\lambda_1^{\omega-1})$ , we carry out  $m-k$  QR-Implicitly shifted as in (2.b) of the algorithm 2 and set  $m=m+\text{fix}(k/3)$  and go

to step (3.c); else set  $\hat{\lambda}_1^{\omega-1} = \lambda_1^\omega$  and continue.

f/ Apply implicit shifts to obtain an Arnoldi Factorization of length  $k$ .

In step1, the algorithm calculates the initial sets: the  $k$  Arnoldi factorization  $AV_k^0 = V_k^0 H_k^0 + f_k^0 e_k^T$ . And the corresponding eigenpairs  $(\lambda_i^0, y_i^0)$  of  $H_k^0$  on step 2, the residual vector satisfies  $r_i^0 = f_k^0 e_k^T y_i^0$ .

In the main loop (step 3); the routine is carried from  $\omega=1$  up to the convergence.  $\Xi$  is the cosine of the acute angle between  $\eta_i^\omega$

and  $\eta_i^{\omega-1}$  with  $\eta_i^\omega = \eta_i^\omega - \eta_i^{\omega-1}$ . And  $\Xi_i = \frac{(\eta_i^\omega)^T \eta_i^{\omega-1}}{\|\eta_i^{\omega-1}\| \|\eta_i^\omega\|}$

The step (3.e) is important in the fact that if the damping ration  $\zeta(\lambda_1^\omega) > \zeta(\lambda_1^{\omega-1})$ , then we carry out  $m-k$  implicit QR steps and reconstruct the Arnoldi factorization with a larger dimension  $m$  ( $m < m_{max}$ )  $m_{max}$  is the greatest possible value of the Krylov subspace, the default value is  $\min(\max(3k-\text{fix}(k/2), 19, n))$ , where  $\text{fix}(p)$  rounds  $p$  to the nearest integer towards 0,  $\text{rem}(P, Q)$  denotes the remainder in the division of P by Q

## 4. Results of test cases.

In this part, we consider two benchmark systems, which the New England New York 39-bus, 16-machines system and the IEEE 145-bus, 50-machines system. The details both systems are given in Table1.

TABLE 1: DESCRIPTION OF TEST SYSTEMS

	System1	System2
No. of Buses	68	145
No. of Generators :	16	50
- detailed model	16	7
- classical model	0	43
Total No. of states	207	235

### 3.1. System1

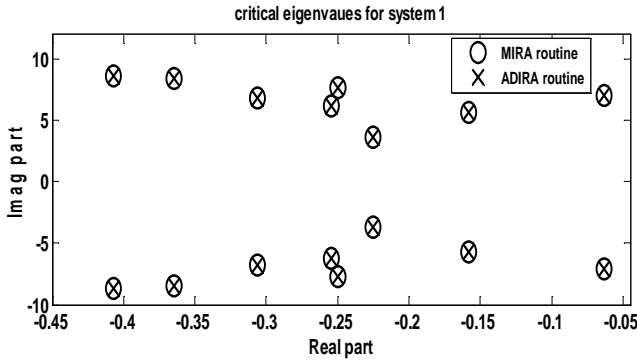
The proposed power system is the New England and New York interconnected system. For this system, generators are modelled using a IV-order model. The full dynamic data of the system can be found in [6] The calculation of the whole spectrum was first done by using the QR-Routine. Setting as 5% the critical damping ratio value, 8 critical oscillatory modes are identified Table2.

TABLE 2: OSCILLATORY MODES FOR SYSTEM1

Mode Num	Eigenvalues	damping	Frequency
1	$-0.0633 \pm j 7.0777$	0.89%	1.127
2	$-0.1583 \pm j 5.6834$	2.78%	0.905
3	$-0.2496 \pm j 7.6788$	3.25%	1.223
4	$-0.2546 \pm j 6.2323$	4.08%	0.992
5	$-0.264 \pm j 7.672$	3.50%	1.221
6	$-0.3649 \pm j 8.4667$	4.31%	1.348
7	$-0.3063 \pm j 6.8011$	4.50%	1.083
8	$-0.4073 \pm j 8.6986$	4.68%	1.385

Comparison was then made by calculating the few sought critical modes by our proposal MIRA and ADIRA algorithms.

Both of algorithms converges to same approximate



values as shown on figure 1

Fig. 1. Sought eigenvalues for system 1 calculated by MIRA and ADIRA

It's clear from figure 1 than both algorithms converge to the same approximate values; in the other hand, a comparison is also done between the results from QR and ADIRA algorithms as shown in table 3

TABLE 3 COMPARISON RESULTS FOR SYSTEM1

Mode Num	Critical Eigenvalues with QR	Sought Eigenvalues with ADIRA
1	$-0.0633 \pm j 7.0777$	$-0.0633 \pm j 7.0773$
2	$-0.1583 \pm j 5.6834$	$-0.1586 \pm j 5.685$
3	$-0.2496 \pm j 7.6788$	$-0.2488 \pm j 7.677$
4	$-0.2546 \pm j 6.2323$	$-0.2541 \pm j 6.2333$
5	$-0.264 \pm j 7.672$	$-0.2645 \pm j 7.6727$
6	$-0.3649 \pm j 8.4667$	$-0.3644 \pm j 8.464$
7	$-0.3063 \pm j 6.8011$	$-0.3060 \pm j 6.8013$
8	$-0.4073 \pm j 8.6986$	$-0.4072 \pm j 8.697$

#### 4.2 System2

We consider IEEE 145-bus, 50-machine test case system; the dynamic data are given by [15] [7]. This system is formed by 145 buses, 453 line/transformers, and 50 machines, among them 7 are modeled through a VI-order model and are equipped with IEEE ST1a excitors. The classical model is used for the remaining machines.

As for the first system, a QR routine is carried out to obtain the whole of the spectrum.

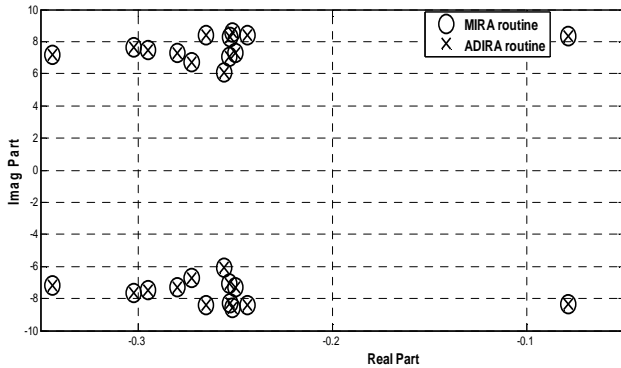
The oscillatory modes are represented on Table 4

TABLE 4. OSCILLATORY MODES FOR SYSTEM2

Mode Num	Eigenvalues	damping %
1	$-0.256 \pm j 6.094$	4.19%
2	$-0.272 \pm j 6.736$	4.04%
3	$-0.253 \pm j 7.074$	3.57%
4	$-0.344 \pm j 7.185$	4.78%
5	$-0.250 \pm j 7.295$	3.43%
6	$-0.292 \pm j 7.471$	3.94%
7	$-0.302 \pm j 7.633$	3.96%
8	$-0.252 \pm j 8.279$	3.05%
9	$-0.079 \pm j 8.362$	0.94%
10	$-0.280 \pm j 7.308$	3.83%
11	$-0.244 \pm j 8.384$	2.91%
12	$-0.265 \pm j 8.340$	3.15%
13	$-0.251 \pm j 8.566$	2.93%

The eigenvalues of interest are calculated from both MIRA and ADIRA algorithms, figure 2 shows the same values found from both methods

Fig. 2. Sought eigenvalues for system 2 calculated by MIRA and ADIRA



From table 5, we can see clearly that the results are the same calculating from both techniques

TABLE 5.COMPARISON RESULTS FOR SYSTEM2

Mode No.	QR- Routine	ADIRA algorithm
1	-0.256 ± j 6.094	-0.2558 ± j 6.0937
2	-0.272 ± j 6.736	-0.2723 ± j 6.7363
3	-0.253 ± j 7.074	-0.2530 ± j 7.0737
4	-0.344 ± j 7.185	-0.3441 ± j 7.1846
5	-0.250 ± j 7.295	-0.2500 ± j 7.2949
6	-0.292 ± j 7.471	-0.2947 ± j 7.4708
7	-0.302 ± j 7.633	-0.3021 ± j 7.6326
8	-0.252 ± j 8.279	-0.2525 ± j 8.2790
9	-0.079 ± j 8.362	-0.0788 ± j 8.3620
10	-0.280 ± j 7.308	-0.2798 ± j 7.3078
11	-0.244 ± j 8.384	-0.2437 ± j 8.3837
12	-0.265 ± j 8.340	-0.2651 ± j 8.3998
13	-0.251 ± j 8.566	-0.2514 ± j 8.5658

#### 4.3 CPU time comparison

The comparison of the CPU times for the above eigenvalue calculations between MIRA and ADIRA are given in Table 5. The computations have been performed using MatLab with a tolerance  $10^{-10}$  for convergence

TABLE 6 .PU TIME COMPARISON

system	QR	MIRA	ADIRA	Residue
1	$1.80 \times 10^{-01}$	$1.15 \times 10^{-2}$	$7.27 \times 10^{-3}$	$10^{-10}$
2	$3.4 \times 10^{-2}$	$2.5 \times 10^{-2}$	$9.72 \times 10^{-03}$	$1.13 \times 10^{-12}$

#### 4.4 Discussion and general comments

From Tables 2 to 5, it is clear that both methods give practically identical results for both test systems. The complete analysis using the QR routine determines

the whole spectrum, but for the small signal stability analysis, only few critical eigenvalues are of interest.

Therefore, it's more interesting to use Krylov subspaces, this takes fewer memory place and faster computational time than the QR routine.

The first step in our work was to modify the well-known algorithm which is the Implicitly Restarted Arnoldi Algorithm in such way to find directly the dominant oscillatory modes of the system by introducing the damping ratio as a criteria for selecting the wanted and the shifts eigenvalues in the algorithm.

The following step was to accelerate the algorithm by coupling it with a switching dynamic procedure as proposed on [9]. This will reduce the computational by reducing the orthogonalization steps.

The numerical results show clearly that the ADIRA becomes faster when coupled with the dynamic switching routine.

#### 5. Conclusion.

We have developed an adaptive dynamique strategy for accelerating the convergence of the implicitly restarted Arnoldi method for the small signal stability analysis.

The test system demonstrate the efficiency of the proposed method by finding the oscillatory modes of the system with lower computational time for convergence, as the method is designed to reduce the number of orthogonalization steps.

We believe that these features can be very helpful for large power system small signal stability analysis, the technique may save memory space and computation time.

#### References

- [1] P.Kundur, *Power System control and stability*, Mc Graw Hill, NY 1994.
- [2] P.Kundur, G.J Rogers, D.Y Wong, L.Wang, M.G.Lauby, *A comprehensive computer program package for small signal stability of power systems*, IEEE trans, Vol.5, N.5, pp1076-1083, Nov1990.
- [3] D.Wong, G.Rogers, B.Porraeta, and P.Kundur, *Eigenvalue analysis of very Large Power Systems*, IEEE Trans, vol.pwrs-3, pp 472-480, May 1988.
- [4] Y.Saad, *Numerical Methods For Large Eigenvalue problems*, Halsted Press, New York, 1992.
- [5] G.Anderson, *Modelling and Analysis of Electric Power Systems*, EEH, power systems laboratory, March 2006.
- [6] G.Rogers, *Power system Oscillations*, Kluwer Academic Publishers, 2000
- [7] S.Liu, *Assessing placement of controllers and nonlinear behaviour of Electrical power system using normal form information*, PhD thesis, 2006.

- [8] D.C.Sorensen *Implicitly restarted Arnoldi/Lanczos methods for Large eigenvalues calculations*, ICASE report N° 96-40 1996
- [9] K.Dookhitram, R.Boojhawon,M.Bhuruth, *A new method for accelerating Arnoldi Algorithms for Large scale eigenproblems*, Elsevier, Mathematics and computers in simulation, 387-401, july 2009.
- [10] G.Rogers, *Power System Toolbox Ver. 2.0: Dynamic Tutorial and Functions*, Cherry Tree Scientific Software. Colborne, ON, Canada, 1999
- [11] G.W Stewart, *A Krylov-Schur algorithm for large problems*, siam J.matrix anal Appl, 2001
- [12] G.W Stewart, *Matrix Algorithms, Vol II: Eigensystems*, Siam 2001
- [13] R.B.Lehoucq, D.C.Sorensen, C.Yang, *ARPACK User's guide:Solution of Large Scale Eigenvalue Problems With Implicitly Restarted Arnoldi Methods*, SIAM Philadelphia, PA 1998
- [14] I.S. Duff, J.A. Scott, *Computing selected eigenvalues of large sparse unsymmetric matrices using subspace iteration*, ACM Trans. Math. Software 19 (1993) 137–159
- [15] [http://psdyn.ece.wisc.edu/IEEE\\_benchmarks](http://psdyn.ece.wisc.edu/IEEE_benchmarks)

#### Appendix A.

##### Dynamic Switching Procedure

Input  $\Xi_l, m, k, m_{min}, m_{max}, \omega, d^{\omega}, tol$

Output  $m, m_{max}$ ,

if ( $|\Xi_l| < 1.0 - tol$ ),

if ( $k > 7$ ),

if ( $rem(\omega, 6) \neq 0$ ), then  $m = m + 1$ ; end

if ( $m \geq m_{max}$ ) then  $m = m_{min}$ ; end

else

if ( $rem(\omega, 6) \neq 0$ ), then  $m = m - 1$ ; end

if ( $m < m_{min}$ ) then  $m = m_{max} - 1$ ; end

end

else

if ( $\omega = 1$ )

if ( $k > 7$ ),  $m = m_{min}$ ; else  $m = m_{max} - 1$ ;

end

end

if ( $m - 1 > m_{min}$ ) and ( $m - 1 \geq d^{\omega} + 2$ )

if ( $k > 7$ )

$m = m - 1$

else

if ( $rem(\omega, 6) = 0$ ,

$m = m - 1$ ; else  $m = m - 3$ ; end

end

else

if ( $m_{max} - 1 \geq d^{\omega} + 2$ ),  $m = m_{max} - 1$ ;

elseif ( $m_{max} - 1 = d^{\omega} + 1$ ),  $m_{max} = m_{max} + 1$ ;

$m = m_{max} - 1$ ;

elseif ( $m_{max} - 1 = d^{\omega}$ ),  $m_{max} = m_{max} + 2$ ;  $m =$

$m_{max} - 1$ ;

end

end

end

with:

$$\Xi_l = \frac{(\eta_1^{\omega})^T \eta_1^{\omega-1}}{\|\eta_1^{\omega-1}\| \|\eta_1^{\omega}\|}$$

the parameter  $m$ , the dimension of  $K_m(A, v^{(\omega-1)})$

one step of reorthogonalization is carried out at each cycle  $\omega$  so that ort is equivalent to

$$\sum_{i=1}^m [m(m+1) - d^{\omega} (d^{\omega} + 1)]$$

in this paper  $d^1 = 0$  and for  $\omega = 2, 3, \dots, d^{\omega}$  is chosen dynamically starting from  $d^2 = k$ , it represents the number of ritz vectors that are kept at each restarted cycle  $\omega$ .

$m_{min}$ : the minimum size of the Krylov subspace

$m_{max}$ : the maximum size of the Krylov subspace

tol: the tolerance parameter