

SIMULATION OF SUSPENSION SYSTEM WITH SLIDING MODE OBSERVER AND CONTROL

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Abstract: This paper presents a Sliding Mode Observer (SMO) design for quarter car model of an active suspension system. Sliding surface control law using the estimated states is applied to the quarter car model. Control using estimated states is shown to yield ride quality improvement. The performance of Sliding Mode Control (SMC) with SMO is compared to an optimal controller with Luenberger observer. Simulation results show the superiority of the sliding mode observer and control.

Key words: Vehicle suspension, observer, sliding mode control, ride comfort.

1. Introduction

Active suspension systems have been introduced in early 70's to minimize the vertical forces transmitted to the passengers to enhance the ride comfort. The basic idea in active control of suspension is to use an active element (the actuator) to apply a desired force between the vehicle body and wheel axle. This desired force is computed by the vehicle control unit to achieve certain performance objectives under external disturbances, such as passenger's comfort under road imperfections [1]. The main advantage of an active suspension is its adaptation potential where the suspension characteristics can be adjusted according to the profile of the road being traversed.

Various control strategies such as optimal state-feedback [2-4], back stepping method [5], fuzzy control [6-8], sliding mode control [9, 10] and fuzzy sliding mode control [11] have been proposed in the past to control the active suspension system. Sliding Mode Control (SMC) has been applied to non-linear systems and it is considered as an effective approach for the control of the systems with uncertainties [12]. In SMC full state feedback control structure is used with an addition of a switching term that is aimed to cancel the effects of uncertainties [13]. The method of controlling a quarter-car hydraulic active suspension system using SMC technique that guarantees the stability of the existence of a sliding

mode and also the stability of the overall system has been developed [14]. Practical implementations require controllers that are robust to uncertainties and disturbances. High order sliding mode observer to estimate the vertical force are presented and tested on a test bench which is a static vehicle excited vertically by hydraulic jacks [15]. Dixit and Buckner presented a robust, nonlinear observation and control namely sliding mode observation and control to a semi active vehicle suspension using model reference approach [16].

The objective of this paper is to use Sliding Mode Control law for an active suspension system which in turn requires the estimation of state variables by a suitable observer. Sliding mode observer design by Dixit and Buckner is used in this paper. Performance of active suspension system using combined Sliding Mode Observer and Control (SMOC) is compared with optimal controller with Luenberger observer.

Simulation results show the effectiveness of the proposed control scheme in suppressing the undesirable effects of the suspension system. This paper is organized into the following sections. Quarter-car model with actuator dynamics is briefly explained in section 2. SMC design is briefed in section 3. SMO design is presented in section 4. Simulation results are discussed and presented in section 5. Finally, the last section concludes the paper.

2. Active suspension model

2.1 Quarter car model

Quarter car models are very often used for suspension analysis and design, because they are simple yet capture many important characteristics of the full model [6]. A two degree of freedom "quarter-car" automotive suspension system is shown

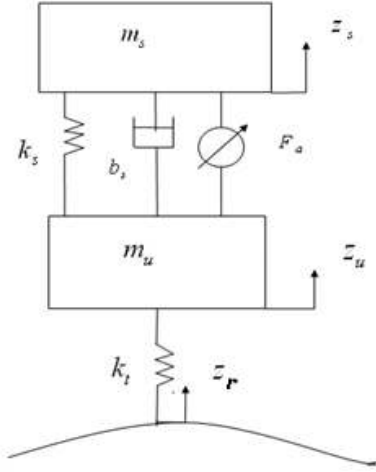


Fig.1. Quarter car model

in Figure 1. It represents the automotive system at each wheel i.e. the motion of the axle and of the vehicle body at any one of the four wheels of the vehicle. The suspension is shown to consist of a spring k_s , a damper b_s and an active force actuator F_a . The active force can be set to zero in a passive suspension.

The sprung mass m_s represent the quarter car equivalent of the vehicle body mass. An unsprung mass m_u represents the equivalent mass due to axle and tyre. The vertical stiffness of the tyre is represented by the spring k_t . The variables z_s , z_u and z_r represent the vertical displacements from static equilibrium of the sprung mass, unsprung mass and the road respectively. Equations of motion of the two degree of freedom quarter car suspension is given by

$$\begin{aligned} m_s \ddot{z}_s &= F_a - k_s(z_s - z_u) - b_s(\dot{z}_s - \dot{z}_u) \\ m_u \ddot{z}_u &= k_s(z_s - z_u) + b_s(\dot{z}_s - \dot{z}_u) + k_t(z_r - z_u) - F_a \end{aligned} \quad (1)$$

It is assumed that the suspension spring stiffness and tyre stiffness are linear in their operating ranges and that tyre does not leave the ground. It is convenient to define the state vector as follows when writing these equations in state space

$$[x_1, x_2, x_3, x_4] = [(z_s - z_u), \dot{z}_s, z_u - z_r, \dot{z}_u]$$

where x_1 is suspension deflection, x_2 is the sprung mass velocity, x_3 is tyre deflection and x_4 is the unsprung mass velocity. The state space model of the quarter car active automotive suspension system can be written as

$$\begin{aligned} \dot{X}(t) &= Ax(t) + Bu(t) + Gw(t) \\ Y &= Cx(t) \end{aligned} \quad (2)$$

where the state matrix A , control vector B , disturbance input vector G and the output matrix C are given as

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-k_s}{m_s} & \frac{-b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & \frac{-k_t}{m_u} & \frac{-b_s}{m_u} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1/m_s \\ 0 \\ -1/m_u \end{bmatrix}$$

$$G = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad \text{and} \quad C = [1 \quad 0 \quad 0 \quad 0]$$

with x being the system state, u being the system input, $w(t)$ is the road disturbance. Equation (2) shows that the disturbance input is not in phase with the system input, i.e. $\text{rank}[B] \neq \text{rank}[B, G]$, therefore the system suffers from mismatched condition. Hence, the controller must be robust enough to overcome the mismatched condition so that the disturbance would not have significant effect on the performance of the system. Hence equation (1) can be written as

$$\dot{X}(t) = Ax(t) + Bu(t) + f(x, t) \quad (3)$$

where $f(x, t)$ are the uncertainties with mismatched condition.

The following assumptions are taken as standard

Assumption 1.

The state vector $x(t)$ is fully observable.

Assumption 2.

There exists a known positive constant β such that $\|f(x, t)\| \leq \beta$ where $\|\bullet\|$ denotes the standard Euclidean norm.

Assumption 3.

The pair (A, B) is controllable and the input matrix B has full rank.

3. Controller Design

High-performance control of the vehicle suspensions is complicated by nonlinearities and uncertainties in the system dynamics and by the need for accurate state information. For these reasons, practical implementations require nonlinear controllers and observers that are robust to uncertainties and disturbances. In this section, sliding mode controller is developed for the nonlinear, active suspension system. Ideally, this controller will be robust to parameter variations and unmodelled dynamics. SMC requires accurate knowledge of the states that are not readily measurable. In this paper, a robust observer is developed to provide state estimation using only measured suspension deflection as an input. To prove the effectiveness of sliding mode observer and control the performance of the active suspension of quarter car model is compared with LQR control and Luenberger observer.

3.1 LQR controller

As an optimal controller, the Linear Quadratic Regulator (LQR) is used to find the actuator force, which minimizes a Performance Index that includes the acceleration of the body, suspension deflection and tyre deflection. To design a state feedback controller, the Performance Index is defined as

$$J = \frac{1}{2} \int (X^T Q X + U^T R U) dt \quad (4)$$

where the matrix Q is symmetric positive semi-definite and R is symmetric positive definite. Then the optimal linear feedback control law is obtained as

$$\begin{aligned} u &= -Kx \\ K &= R^{-1} B^T P \end{aligned} \quad (5)$$

where $K=[K_1, K_2, K_3, K_4]$ is the Kalman gain and P is given by the solution of algebraic Riccati equation

$$-P = A^T P + PA - PBR^{-1}B^T P + Q \quad (6)$$

The optimal actuator force represented as a function of states

$$u = -K_1 x_1 - K_2 x_2 - K_3 x_3 - K_4 x_4 \quad (7)$$

Obviously, when the system state is not directly measured, it is reconstructed via an asymptotic observer, and the above control law is replaced by

$$u = -K\tilde{x} \quad (8)$$

where \tilde{x} is the state estimate.

3.2 Sliding Mode Control

SMC technique is designed to drive the state

trajectory towards the sliding surface. Sliding mode controller design starts with the design of the sliding surface that ensures the stability of the system. It is proved mathematically that the reachability condition of the proposed controller is achieved if the proposed theorem is satisfied to ensure that the state trajectory slides onto the sliding surface and remains thereafter.

The first step in SMC is to design sliding surface.

Let the time varying switching surface, $\sigma(x, t) = 0$ in the state space:

$$\sigma(x, t) = Sx \quad (9)$$

where $S = [s_1, s_2, \dots, s_{n-1}, 1]$ is a strictly positive real constant and

$$\begin{aligned} \sigma &= s_1 x_1(t) + s_2 x_2(t) + \dots + s_{n-1} x_{n-1}(t) \\ &+ x_n(t) = 0 \end{aligned} \quad (10)$$

To evaluate stability, the Lyapunov function is taken as

$$V = \frac{1}{2} \sigma^2(t) \quad (11)$$

The existence condition for the sliding mode is satisfied if

$$\dot{V} = \sigma \dot{\sigma} < 0 \quad (12)$$

The necessary condition for the state trajectory to stay on the sliding surface is

$$\dot{\sigma}(x, t) = 0 \quad (13)$$

and the equivalent control (u_{eq}) is obtained from (12)

$$\dot{\sigma}(x, t) = S\dot{x} = 0 \quad (14)$$

$$u_{eq} = -(SB)^{-1}[SAx(t) + Sf(x, t)] \quad (15)$$

Substituting (15) in (3) give the equivalent dynamics equation of the system in sliding mode as follows

$$\begin{aligned} \dot{X}(t) &= Ax(t) - B((SB)^{-1}[SAx(t) + Sf(x, t)]) \\ &+ f(x, t) \end{aligned}$$

$$\begin{aligned} \dot{X}(t) &= [A - B(SB)^{-1}SA]x(t) + \\ &[I_n - B(SB)^{-1}S]f(x, t) \end{aligned} \quad (16)$$

During sliding mode, system with uncertainties and mismatched condition is stable provided following theorem is satisfied [17].

Theorem 1

The uncertain system described in (3) is bounded stable on the sliding surface, $\sigma(x, t) = 0$ if

$$\|\tilde{F}(t)\| \leq \beta_1 = \|I_n - B(SB)^{-1}S\|\beta \quad (17)$$

Proof :

$$\text{Let } \tilde{A} = [A - B(SB)^{-1}SA]$$

$$\tilde{F}(t) = [I_n - B(SB)^{-1}S]f(x, t)$$

and (16) is rewritten as

$$\dot{X}(t) = \tilde{A}x(t) + \tilde{F}(t) \quad (18)$$

Lyapunov function for the system is chosen as

$$V(t) = x^T(t)Px(t) \quad (19)$$

Taking derivative of $V(t)$ and substituting (16)

$$\begin{aligned} \dot{V} &= x^T(t)[\tilde{A}^T P + P\tilde{A}]x(t) + \tilde{F}^T(t)Px(t) \\ &\quad + x^T(t)P\tilde{F}(t) \\ &= -x^T(t)Qx(t) + \tilde{F}^T(t)Px(t) \\ &\quad + x^T(t)P\tilde{F}(t) \end{aligned} \quad (20)$$

where P is the solution of $\tilde{A}^T P + P\tilde{A} = -Q$ for a given positive definite symmetric matrix Q.

Equation (20) is reduced to

$$\dot{V} = -\lambda_{\min}(Q)\|x(t)\|^2 + 2\beta_1\|P\|\|x(t)\| \quad (21)$$

Since $\lambda_{\min}(Q) > 0$, $\dot{V}(t) < 0$ for all t and $x \in B^c(\eta)$, where $B^c(\eta)$ is the complement of the closed ball $B(\eta)$,

centered at $x = 0$, radius $\eta = \frac{2\beta_1\|P\|}{\lambda_{\min}(Q)}$.

Hence the system is bounded stable.

Theorem 1 shows if the condition (17) is satisfied, the system is bounded stable in the presence of mismatched conditions.

Control law

The objective is to design a control scheme that drives the state trajectories of the system defined in (3) towards the sliding surface and remains in the stable manifold thereafter. The control law that satisfies the sliding mode condition for the system (3) is given as [11]

$$u = -(SB)^{-1}[SAx(t) + Sf(x, t) - \rho(SB)^{-1} \frac{\sigma(t)}{\|\sigma(t)\| + \Phi}] \quad (22)$$

where Φ is the boundary layer thickness that is introduced to avoid chattering effect and ' ρ ' is a constant representing the maximum controller output which is specified by the designer.

Theorem 2

The reaching condition of the sliding surface (9) is satisfied if

$$\|A - B(SB)^{-1}SA\|\|x(t)\| \geq \|f(t)\| \quad (23)$$

Proof:

Reaching condition is evaluated as follows:

$$\begin{aligned} \sigma\dot{\sigma}(t) &= \sigma(t)[S(Ax(t) + Bu(t) + f(x, t))] \\ &= \sigma(t)[(SAx(t) + SBu(t) + Sf(x, t))] \end{aligned}$$

Substituting u (t) using (22),

$$\sigma\dot{\sigma}(t) = \sigma(t) \left[-\rho \frac{\sigma(t)}{\|\sigma(t)\| + \Phi} \right] \quad (24)$$

The sliding condition is established if $\rho > 0$. The theorem ensures that the proposed control law drives the system state trajectory onto the sliding surface and remains on it thereafter. However removal of chattering in SMC has been addressed in literatures [11].

4. OBSERVER DESIGN

The control law discussed in the previous section requires the knowledge of the system state variables. Since all the state variables are not available for measurement except for suspension deflection an appropriate state observer is designed. Figure 2 shows the block diagram of the state feedback controller with observer. The observer is a subsystem to reconstruct the state vector of the plant. The mathematical model of the observer is same as that of the plant, except that an additional term that includes the estimation error to compensate for inaccuracies in matrices A and B and the lack of initial error. The estimation error is the difference between the measured output and the initial estimated state. Thus, the mathematical model of the observer is

$$\dot{\tilde{x}} = (A - k_e C)\tilde{x} + Bu + k_e y \quad (25)$$

where \tilde{x} the estimated state and $C\tilde{x}$ is estimated output. The inputs to the observer are the output y and the control input u . Matrix k_e is the observer gain matrix and is a weighing matrix to the correction term involving the difference between the measured output y and the estimated output $C\tilde{x}$. This term updates the model output and improves the performance of the observer.

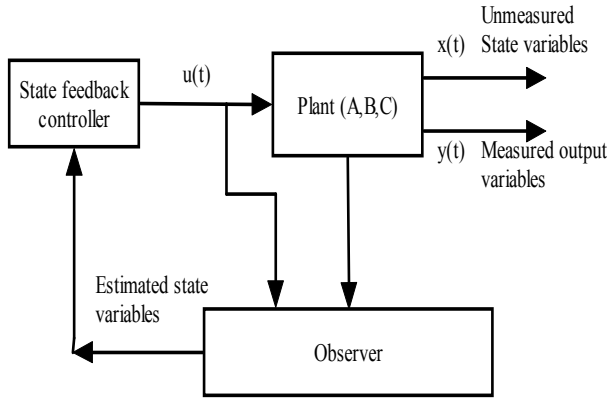


Fig 2. Block diagram of state feedback controller with observer

4.1 Luenberger Observer

Consider state model described by (2) where the matrices A , B and C are parameters of the state space model. The well-known Luenberger Observer is given as

$$\dot{\tilde{x}} = A\tilde{x} + Bu + L(y - C\tilde{x}) \quad (26)$$

where

\tilde{x} represents the estimated state vector, L is the Luenberger observer matrix.

The estimation error is

$$e = x - \tilde{x} \quad (27)$$

The error dynamic equation is given by

$$\dot{e} = (A - LC)e \quad (28)$$

The estimation error will converge to zero if $A - LC$ has all its eigen values in the left-half plane. The observer design refers to the selection of the gain matrix L , using the pole placement method. Conventional observer cannot guarantee global asymptotic stability, robustness or convergence in the presence of plant nonlinearities, parametric variations and disturbances. More over separation principle cannot be applied for nonlinear application and hence closed loop stability cannot be guaranteed.

4.2 Sliding Mode Observer

SMO enable robust, accurate estimation of system states for nonlinear, uncertain systems where conventional observers or Kalman filters perform poorly [18]. Since the sliding mode observer is not dependent on the plant and only the output y is needed and it is a model free observer. This section describes the design of a sliding mode observer,

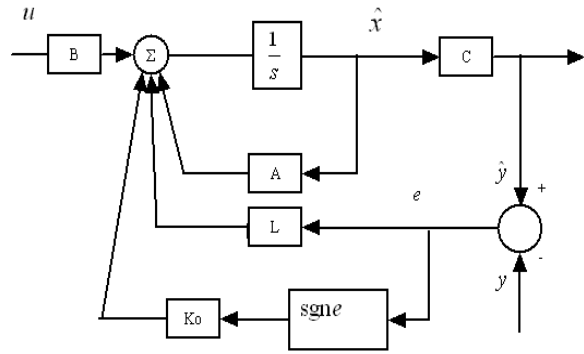


Fig. 3 Sliding mode observer structure

which is based on a standard Luenberger observer but includes a discontinuous performance term for robustness.

The structure of the SMO is illustrated in the Figure 3.

$$\dot{\tilde{x}} = A\tilde{x} + Bu + L(y - C\tilde{x}) + K_0 \text{sgn}(y - C\tilde{x}) \quad (29)$$

where K_0 is the SMO switching gain matrix.

$$e = y - \tilde{y} = C(x - \tilde{x}) \quad (30)$$

$$\dot{e} = C(\dot{x} - \dot{\tilde{x}}) = (A - LC)e + f(x, t) - K_0 \text{sgn}(e) \quad (31)$$

Because AC is observable there exists L such that $(A - LC)$ is stable. K_0 is obtained using LQ technique or Nyquist criteria approaches. To avoid chattering, the performance term $K_0 \text{sgn}(y - C\tilde{x})$ can be constrained to be continuous as long as the system states lie within a boundary layer defined by error limits between the measured and estimated output states

$$x_p = K_0 \text{sat} \left[\frac{y - C\tilde{x}}{\Phi} \right] \quad (32)$$

where Φ is the boundary layer width. When the observer error exceeds the boundary layer width, the performance term becomes discontinuous. The saturation term is defined to be

$$\text{sat} \left[\frac{y - \tilde{y}}{\Phi} \right] = \begin{cases} \frac{y - \tilde{y}}{\Phi} & \text{for } |y - \tilde{y}| \leq \Phi \\ \text{sgn}(y - \tilde{y}) & \text{for } |y - \tilde{y}| > \Phi \end{cases} \quad (33)$$

If $f(x,t)$ is bounded and the observer gain satisfies the condition $K_0 > \beta$, error equation is

asymptotically stable. i.e. $\lim_{t \rightarrow \infty} e = 0$. However for an optimal and SMC, full state measurement is assumed instead of partial states.

5. Simulation Results

The mathematical model defined in equation (2) and the sliding mode controller defined in equation (13) is simulated using Matlab and simulink. Bump input is considered as the disturbance to the system. Bump input has been used in the literature and is of the form [9]

$$r(t) = \begin{cases} a(1 - \cos(8\pi t))/2 & \text{if } 1.0 \leq t \leq 1.25 \text{ and } 3.0 \leq t \leq 3.25 \\ 0 & \text{otherwise} \end{cases} \quad (34)$$

where a denotes the bump amplitude. The parameters of the quarter car model are listed as follows [6]

| | | |
|--------------------------------|---|-------------|
| Sprung Mass (m_s) | - | 240 kg |
| Unsprung Mass (m_u) | - | 36 kg |
| Damper coefficient (b_s) | - | 980 Ns/m |
| Suspension Stiffness (k_s) | - | 16,000 N/m |
| Tyre Stiffness (k_t) | - | 160,000 N/m |

In the design of LQR controller, weighing matrices are selected as $Q = \text{diag}(q_1, q_2, q_3, q_4)$ where $q_1=1000$, $q_2=2 \times 10^5$, $q_3=1000$, $q_4=1000$ and $R = [0.001]$. Matrix K is calculated as $[310 \ 43850 \ -178030 \ -610]$ Closed loop poles are given by -2.0218 , $-0.1419+0.62351i$, $-0.1419-0.62351i$ and -0.0037 . Hence the system is stable. L matrix designed using pole placement Technique. Observer poles chosen as $[-20 \ -15 \ -20 \ -15]$; $K_o = [20 \ 20 \ 15 \ 15]$; $\Phi = 0.04$.

Figure 4a shows that SMO estimate of suspension deflection is nearly indistinguishable from the actual plant output. Fig 4b shows the error convergence of the SMO.

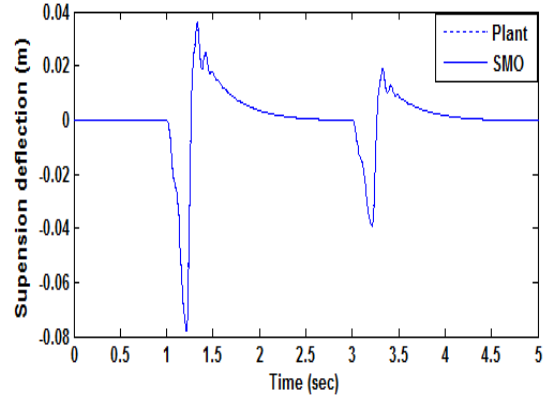


Fig. 4a Estimated suspension deflection

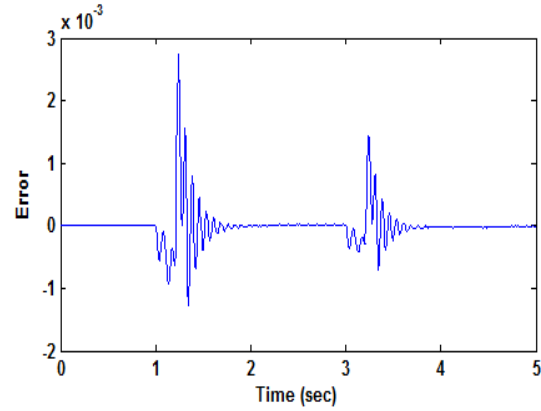
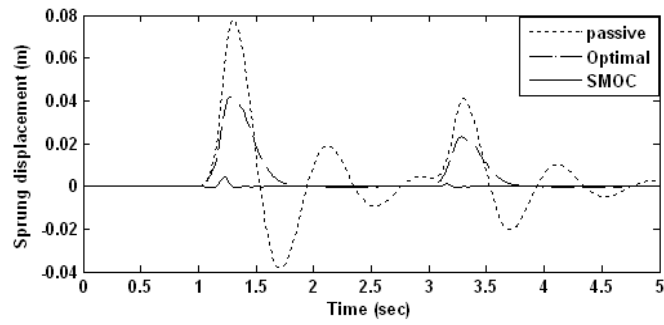
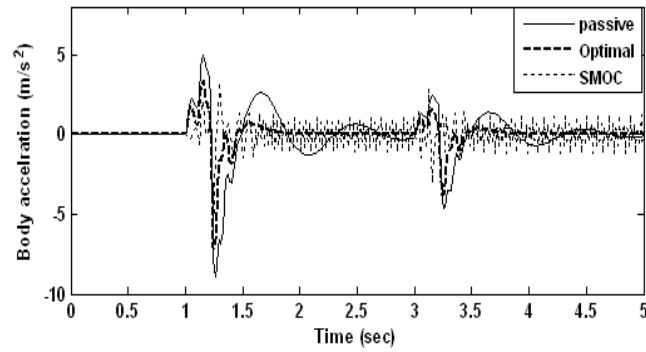


Fig. 4b Error convergence of SMO

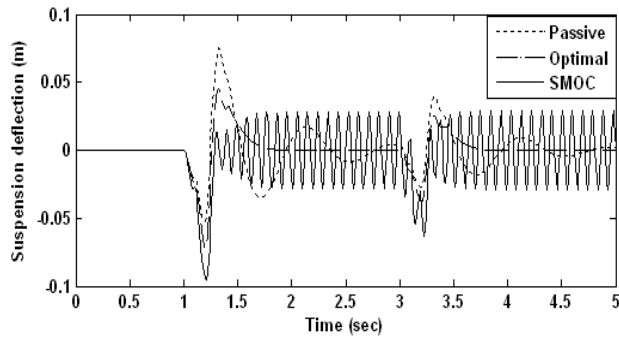
Figure 5a-d shows the simulation results for the passive, optimal controller and SMOC based active suspension system for the bump road input. Figure 5a shows the sprung mass displacement very much reduced by the proposed control scheme and the maximum displacement is less than a cm. Figure 5b shows that the body acceleration is reduced by 50% compared to passive and 9% compared to optimal controller by the proposed control scheme. Figure 5c indicates that the suspension deflection controlled by SMOC is slightly more than the optimal controller. Figure 5d shows the tyre deflection is 50% more for SMOC scheme.



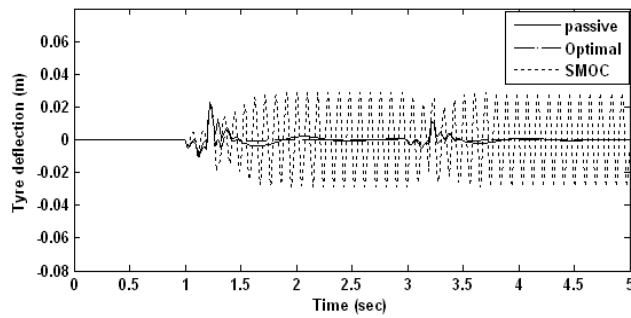
(a)



(b)



(c)



(d)

Fig.5. Sprung mass displacement (a), Body Acceleration (b), Suspension deflection(c) and Tyre deflection (d) for dual bump road profile

Table 1

| Control scheme | Sprung mass Displacement 10^{-3} (m) | Suspension deflection 10^{-2} (m) | Body acceleration (m/s ²) | Tyre-deflection 10^{-2} (m) |
|----------------|--|-------------------------------------|---------------------------------------|-------------------------------|
| Passive | 19.74 | 1.792 | 1.595 | 0.2696 |
| optimal | 10.27 | 1.368 | 0.8734 | 0.233 |
| SMOC | 0.4765 | 2.259 | 0.8037 | 0.388 |

The comparison of the controller performances are presented in Table-1 which shows the Root Mean Squares (RMS) of the body acceleration, suspension deflection, body displacement and tyre deflection. The results show that proposed SMO and control scheme performs better in reducing the sprung mass displacement and acceleration thus providing the desired ride comfort. But due to chattering effect of the SMC road handling is best in the case of optimal controller compared to SMC.

6. Conclusion

In this paper the performance of SMO and SMC has been investigated. It has been shown that the SMC with sliding mode observer improved the ride comfort of quarter car active suspension system compared to LQR controller with Luenburger observer control technique. It is also proved from the RMS values of suspension parameters that the SMOC based active suspension provided excellent ride characteristics when compared to existing passive and optimal controller. Road handling is better in optimal controller with conventional observer. In conclusion, the mismatched condition inherent in the system dynamics has been overcome by the proposed SMO and control.

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