OPTIMAL LINEAR PHASE FIR LOW PASS FILTER DESIGN USING CRAZINESS BASED PARTICLE SWARM OPTIMIZATION ALGORITHM

Sangeeta MANDAL, Sakti Prasad GHOSHAL
National Institute of Technology Durgapur
Department of Electrical Engineering, West Bengal, India- 713209
Tel.: +91 343 2547231; Fax: +91 343 2547375; E-mail address: spghoshalnitdgp@gmail.com

Rajib KAR, Durbadal MANDAL
National Institute of Technology Durgapur
Electronics and Communication Engineering Department, West Bengal, India- 713209
Tel.: +91 343 2545021; Fax: +91 343 2547375; E-mail address: durbadal.bittu@gmail.com

***Abstract**: This paper presents an alternative approach for the design of linear phase digital low pass FIR filter using Craziness based Particle Swarm Optimization (CRPSO) approach. FIR filter design is a multi-modal optimization problem. The conventional gradient based optimization techniques are not efficient for digital filter design. Given the filter specifications to be realized, the CRPSO algorithm generates a set of optimal filter coefficients and tries to meet the ideal frequency response characteristics. In this paper, for the given problem, the realization of the optimal FIR low pass filters of different orders has been performed. The simulation results have been compared to those obtained by the well accepted evolutionary algorithm such as Parks and McClellan algorithm (PM), classical particle swarm optimization (PSO). A maximum -28.64 dB stop band attenuation has been achieved compared to -27.17 dB attenuation for the case PSO. The results justify that the proposed optimal filter design approach using CRPSO outperforms PM and PSO, not only in the accuracy of the designed filter but also in the convergence speed and solution quality.*

***Key words**: FIR Filter, PSO, CRPSO, Parks and McClellan Algorithm, Magnitude Response, Convergence, Low Pass Filter*

1. Introduction

A digital filter is a system that performs mathematical operations on samples of discrete-time signals to reduce or enhance certain aspects of the signals. This is in contrast to the other major type of electronic filter, the analog filter, which is an electronic circuit operating on continuous-time analog signals. Digital filters are basic building blocks in many digital signal processing systems. They have wide range of applications in communication, image processing, pattern recognition, etc. There are two major classes of digital filters, namely, finite impulse response (FIR) filters and infinite impulse response (IIR) filters depending on the length of the impulse response [1]. FIR filter is an attractive choice because of the ease in design and stability. By designing the filter taps to be symmetrical about the centre tap position, a FIR filter can be guaranteed to have linear phase. FIR filters are known to have many desirable features such as guaranteed stability, the possibility of achieving exact linear phase characteristic at all frequencies and digital implementation as non-recursive structures. Linear phase FIR filters are also required when time domain features are specified [2]. The most frequently used method for the design of exact linear phase weighted Chebyshev FIR digital filter is the one based on the Remez-exchange algorithm proposed by Parks and McClellan [3]. Further improvements in their results have been reported in [4]. The main limitation of this procedure is that the relative values of the amplitude error in the frequency bands are specified by means of the weighting function, and not by the deviations themselves. Therefore, in case of designing low-pass filters with a given stop band deviation, filter length and cut-off frequency, the program has to be iterated many times [5]. A number of models have been developed for the FIR filter techniques and design optimization methods. This is a thrust research area, aiming at obtaining more general and innovative techniques that are able to solve or optimize new and complex engineering problems [6]. In some cases, such initiatives were successful and proven to exhibit better performance than the conventional approaches. However, there are few drawbacks associated to these methods, e.g., increased computational cost and non-existence of theoretical proof of convergence to global optimum in sufficiently general conditions. Consequently, there is a need to explore for some more pervasive methods to overcome such drawbacks. Different heuristic optimization algorithms such as simulated annealing algorithms [7], genetic algorithm (GA) [8], artificial bee colony algorithm [9], etc. have been widely used for the synthesis of design methods capable of satisfying constraints which would be unattainable. When considering global optimization methods for digital filter design, the GA seems to have attracted considerable attention. Filters designed by GA have the potential of obtaining near global optimum solution [8]. Although standard GA (also known as Real Coded GA (RGA)) shows a good performance for finding the promising regions of the search space, they are inefficient in determining the global optimum in terms of convergence speed and solution quality. The approach detailed in this paper takes the advantage of the power of the stochastic global optimization technique called particle swarm optimization. Although the algorithm is adequate for applications in any kind of parameterized filters, the authors have chosen to focus on real-coefficient FIR low pass filters, in view of their importance in engineering practice. Particle Swarm Optimization (PSO) is an evolutionary algorithm developed by Kennedy and Eberhart in 1995 [10-11]. Several attempts have been made towards the optimization of the FIR Filter [12] [20] and in other areas also [13], using PSO algorithm. The PSO is simple to implement and its convergence may be controlled via few parameters. The limitations of the conventional PSO are that it may be influenced by premature convergence and stagnation problem [14-15]. In order to overcome these problems, the PSO algorithm has been modified in this paper and is employed for FIR filter design.

This paper describes an alternative technique for the FIR low pass digital filter design using Craziness based Particle Swarm Optimization Technique (CRPSO). CRPSO algorithm tries to find the best coefficients that closely match the ideal frequency response. Based upon the improved PSO approach, this paper presents a good and comprehensive set of results, and states arguments for the superiority of the algorithm. Simulation results demonstrate the effectiveness and better performance of the proposed designed method.

The rest of the paper is arranged as follows. In section II, the FIR filter design problem is formulated. Section III briefly discusses on the algorithm of classical PSO and the CRPSO algorithm. Section IV describes the simulation results obtained for low pass FIR digital filter using PM algorithm, classical PSO and the proposed CRPSO approach. Finally, section V concludes the paper.

2. Low Pass FIR Filter Design

A digital FIR filter is characterized by,

$$H(z) = \sum_{n=0}^{N} h(n) z^{-n}, \text{ n=0, 1… N} \qquad (1)$$

where N is the order of the filter which has (N+1) number of coefficients. h(n) is the filter impulse response. It is calculated by applying an impulse signal at the input. The values of h(n) will determine the type of the filter e.g. low pass, high pass, band pass etc. The values of h(n) are to be determined in the design process and N represents the order of the polynomial function. This paper presents the most widely used FIR with h(n) as even symmetric and the order is even. The length of h(n) is N+1 and the number of coefficients is also N+1. In the algorithm, the individual represents h(n). In each iteration, these individuals are updated. Fitness of each particle is calculated using the new coefficients. In each iteration, this fitness is used to improve the search and result obtained after a certain number of iterations or after the error is below a certain limit is considered to be the optimal result. Because its coefficients are symmetrical, the dimension of the problem reduces by a factor of 2. The (N+1)/2 coefficients are then flipped and concatenated to find the required N+1 coefficients. The least error is used to evaluate the fitness of the individual. It takes the error between the frequency response of the ideal and the actual filter. An ideal filter has a magnitude of one on the pass band and a magnitude of zero on the stop band. So, the error for this fitness function is the difference between the magnitudes of this filter and the filter designed using the evolutionary algorithms PSO and CRPSO. The individuals that have lower error values represent better filters i.e., the filters with better frequency responses.

Various filter parameters which are responsible for the optimal filter design are the stop band and pass band normalized frequencies, the pass band and stop band ripples, the stop band attenuation and the transition width. These parameters are mainly decided by the filter coefficients as is evident from transfer functions in (1). Significance of these parameters in actual filter with respect to ideal filter is illustrated in Fig. 1 [16]. Several scholars have developed algorithms in which N, δ_p, and δ_s are fixed while the remaining parameters are optimized [17]. Other algorithms were originally developed by Parks and McClellan in which N, w_p, w_s, and the ratio δ_p/δ_s were fixed [3].

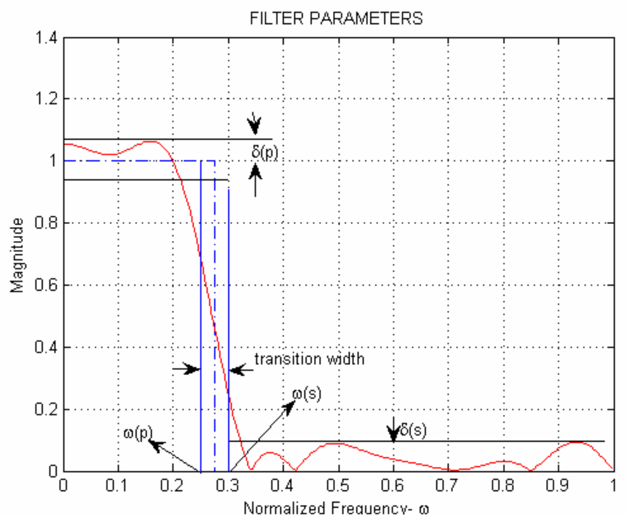

Fig. 1. Filter Parameters.

In this paper, swarm and evolutionary optimization algorithms are applied in order to obtain the actual filter response as close as possible to the ideal response

Now for (1), coefficient vector $\{h_0, h_1 \ldots h_N\}$ is represented in N+1 dimension. The particles are distributed in a D dimensional search space, where D = N+1 for the case of FIR filter.

The frequency response of the FIR digital filter can be calculated as,

$$H\left(e^{jw_k}\right)=\sum_{n=0}^{N} h(n)e^{-jw_k n} \qquad (2)$$

where $H\left(e^{jw_k}\right)$ is the Fourier transform complex vector. This is the FIR filter frequency response. The frequency is sampled in $[0, \pi]$ with N points; the position of each particle in this D dimensional search space represents the coefficients of the transfer function. In every iteration, each particle finds a new position, which is the new set of coefficients. Fitness of all particles is calculated using the new coefficients. These fitnesses are used to improve the search in each iteration, and the result obtained after a certain number of iterations or after the error is below a certain limit is considered to be the final result. Different kinds of fitness functions have been used in different literatures. An error function given by (3) is the approximate error used in Parks–McClellan algorithm for filter design [3].

$$E(\omega)=G(\omega)\left[H_d\left(e^{j\omega}\right)-H_i\left(e^{j\omega}\right)\right] \qquad (3)$$

where $G(\omega)$ is the weighting function used to provide different weights for the approximate errors in different frequency bands, is the frequency response of the desired filter and is given as,

$$H_d\left(e^{j\omega_k}\right)=1 \quad for \quad 0 \le \omega \le \omega_c; \qquad (4)$$
$$=0 \quad \text{otherwise}$$

and $H_i\left(e^{j\omega}\right)$ is the frequency response of the approximate filter [17].

$H_d(\omega)=[H_d(\omega_1), H_d(\omega_2), H_d(\omega_3), \ldots H_d(\omega_K)]^T$ and $H_i(\omega)=[H_i(\omega_1), H_i(\omega_2), H_i(\omega_3) \ldots, H_i(\omega_K)]^T$

The major drawback of PM algorithm is that the ratio of δ_p/δ_s is fixed. To improve the flexibility in the error function to be minimized, so that the desired level of δ_p and δ_s may be specified, the error function given in (5) has been considered as fitness function in many literatures [18].

The error to be minimized is defined as:

$$J_1 = \max_{\omega \le \omega_p}\left(|E(\omega)| - \delta_p\right) + \max_{\omega \ge \omega_s}\left(|E(\omega)| - \delta_s\right) \qquad (5)$$

where δ_p and δ_s are the ripples in the pass band and stop band, ω_p and ω_s are pass band and stop band normalized cut off frequencies, respectively. The error function given in (5) represents the fitness function to be minimized using the evolutionary algorithms. The algorithms try to minimize this error and thus increase the fitness. Since the coefficients of the linear phase filter are matched, meaning the first and the last coefficients are the same; the dimension of the problem is reduced by one-half. By only determining one half of the coefficients, the filter could be designed. This greatly reduced the computational complexity of the algorithms.

3. Evolutionary Techniques Employed

3. A. Particle Swarm Optimization (PSO)

PSO is a flexible, robust population-based stochastic search/optimization technique with implicit parallelism, which can easily handle with non-differential objective functions, unlike traditional optimization methods. PSO is less susceptible to getting trapped on local optima unlike GA, Simulated Annealing, etc. Eberhart and Shi [11] developed PSO concept similar to the behaviour of a swarm of birds. PSO is developed through simulation of bird flocking in multidimensional space. Bird flocking optimizes a certain objective function. Each particle (bird) knows its best value so far (*pbest*). This information corresponds to personal experiences of each particle. Moreover, each particle knows the best value so far in the group (*gbest*) among *pbest*s. Namely, each particle tries to modify its position using the following informations:

• The distance between the current position and the *pbest*,

• The distance between the current position and the *gbest*.

Similar to GA, in PSO techniques also, real-coded particle vectors of population n_p are assumed. Each

particle vector consists of components or sub-strings as required number of normalized filter coefficients, depending on the order of the filter to be designed.
Mathematically, velocities of the particle vectors are modified according to the following equation:

$$V_i^{(k+1)} = w * V_i^{(k)} + C_1 * rand_1 * \left(pbest_i^{(k)} - S_i^{(k)}\right) + C_2 * rand_2 * \left(gbest^{(k)} - S_i^{(k)}\right) \quad (6)$$

where $V_i^{(k)}$ is the velocity of i[th] particle at k[th] iteration; *w* is the weighting function; C_1 and C_2 are the positive weighting factors; $rand_1$ and $rand_2$ are the random numbers between 0 and 1; $S_i^{(k)}$ is the current position of i[th] particle vector at k[th] iteration; $pbest_i^{(k)}$ is the personal best of i[th] particle vector at k[th] iteration; $gbest^{(k)}$ is the group best of the group at k[th] iteration. The searching point in the solution space may be modified by the following equation:

$$S_i^{(k+1)} = S_i^{(k)} + V_i^{(k+1)} \quad (7)$$

The first term of (6) is the previous velocity of the particle vector. The second and third terms are used to change the velocity of the particle. Without the second and third terms, the particle will keep on ‘‘flying’’ in the same direction until it hits the boundary. Namely, it corresponds to a kind of inertia represented by the inertia constant, w and tries to explore new areas.

3. A. i. Craziness based Particle Swarm Optimization (CRPSO)

The global search ability of above conventional PSO is improved with the help of the following modifications. This modified PSO is termed as Craziness based Particle Swarm Optimization Technique (CRPSO).
The velocity in this case can be expressed as follows [19]:

$$V_i^{(k+1)} = r_2 * sign(r_3) * V_i^{(k)} + (1 - r_2) * C_1 * r_1 * \left\{pbest_i^{(k)} - S_i^{(k)}\right\} + (1 - r_2) * C_2 * (1 - r_1) * \left\{gbest^{(k)} - S_i^{(k)}\right\} \quad (8)$$

Where r_1 , r_2 and r_3 are the random parameters uniformly taken from the interval [0,1] and $sign(r_3)$ is a function defined as:

$$sign(r_3) = -1 \quad \text{where} \quad r_3 \le 0.05$$
$$= 1 \quad \text{where} \quad r_3 > 0.05 \quad (9)$$

The two random parameters $rand_1$ and $rand_2$ of (6) are independent. If both are large, both the personal and social experiences are over used and the particle is driven too far away from the local optimum. If both are small, both the personal and social experiences are not used fully and the convergence speed of the technique is reduced. So, instead of taking independent $rand_1$ and $rand_2$, one single random number r_1 is chosen so that when r_1 is large, $(1 - r_1)$ is small and vice versa. Moreover, to control the balance of global and local searches, another random parameter r_2 is introduced. For birds' flocking for food, there could be some rare cases that after the position of the particle is changed according to (7), a bird may not, due to inertia, fly towards a region at which it thinks is most promising for food. Instead, it may be leading toward a region which is in opposite direction of what it should fly in order to reach the expected promising regions. So, in the step that follows, the direction of the bird's velocity should be reversed in order for it to fly back to the promising region. $sign(r_3)$ is introduced for this purpose. In birds' flocking or fish schooling, a bird or a fish often changes directions suddenly. This is described by using a ‘‘craziness’’ factor and is modelled in the technique by using a craziness variable. A craziness operator is introduced in the proposed technique to ensure that the particle would have a predefined craziness probability to maintain the diversity of the particles. Consequently, before updating its position the velocity of the particle is crazed by,

$$V_i^{(k+1)} = V_i^{(k+1)} + P(r_4) * sign(r_4) * v_i^{craziness} \quad (10)$$

where r_4 is a random parameter which is chosen uniformly within the interval [0, 1];
$v_i^{craziness}$ is a random parameter which is uniformly chosen from the interval $\left[v_i^{\min}, v_i^{\max}\right]$; and $P(r_4)$ and $sign(r_4)$ are defined respectively as:

$$P(r_4) = 1 \quad \text{when} \quad r_4 \le P_{cr}$$
$$= 0 \quad \text{when} \quad r_4 > P_{cr} \quad (11)$$

$$sign(r_4) = -1 \quad \text{when} \quad r_4 \le 0.5$$
$$= 1 \quad \text{when} \quad r_4 \le 0.5 \quad (12)$$

where P_{cr} is a predefined probability of craziness and *iter* means iteration cycle number.
The design aim in this paper is to obtain the optimal combination of the filter coefficients, so as to acquire the maximum stop band attenuation with least transition width increment. The values of the parameters used for the CRPSO technique is given in Table I.

Table 1.
CRPSO Parameters

Parameter	Value
Population Size	120
Generation number	1000
C_1 & C_2	1.5
v_i^{min}	1
v_i^{max}	10
p_{cr}	0.3

4. Results and Discussions

4.1. Analysis of Magnitude Response of Low pass FIR filters

The MATLAB simulation has been performed extensively to realize the low pass FIR filter of the orders of 20, 30 and 40. Hence, the lengths of the filter coefficients are 21, 31 and 41, respectively. The sampling frequency has been chosen as f_s = 1Hz. Also, for all the simulations the sampling number is taken as 128. Algorithms are run for 30 times to get the best solutions.
The parameters of the filter to be designed are as follows:

- Pass band ripple (δ_p) = 0.01
- Stop band ripple (δ_s) = 0.001
- Pass band normalized cut-off frequency (ω_p) = 0.40
- Stop band normalized cut-off frequency (ω_s) = 0.45

The best optimized coefficients for the designed filters with the orders of 20, 30 and 40 have been calculated by PM algorithm, PSO and CRPSO and given in Tables II-IV, respectively. Figs. 2 and 3 show the magnitude plot, gain plot, respectively, for the low pass FIR filter of the order of 20. Figs. 4 and 5 show the magnitude plot, gain plot, respectively, for the low pass FIR filter of the order of 30. Figs. 6 and 7 show the magnitude plot, gain plot, respectively, for the low pass FIR filter of the order of 40. From the figures it is evident the proposed filter design approach produces higher stop band attenuation and smaller ripples compared to those of classical PSO for different filter orders. The stop band ripple (or attenuation) in both normalized and in dB for a given transition bandwidth has been shown in Table V. From Table V it may be noted that the CRPSO algorithm can result a maximum stop band attenuation of 20.43 dB, 25.94 dB, 28.64 dB for the filter orders of 20, 30, 40, respectively.

The filters designed by the CRPSO algorithm have sharper transition band responses than those produced by PSO algorithm. For the stop band region, the filters designed by the CRPSO method results in the improved responses than the PSO.

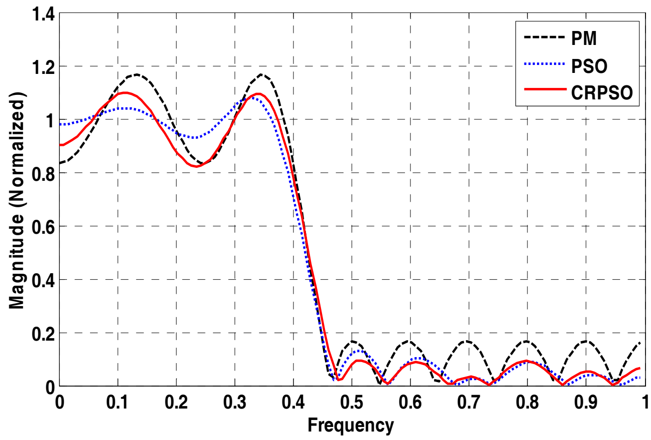

Fig. 2. Normalized Frequency response for the FIR Filter of order 20.

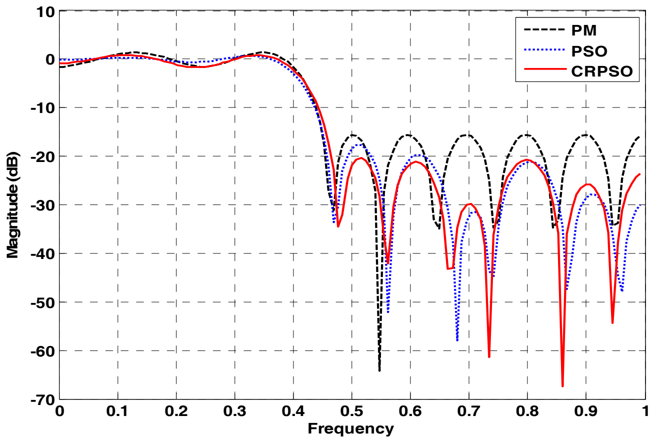

Fig. 3. Gain (dB) Plot of the FIR Filter of Order 20.

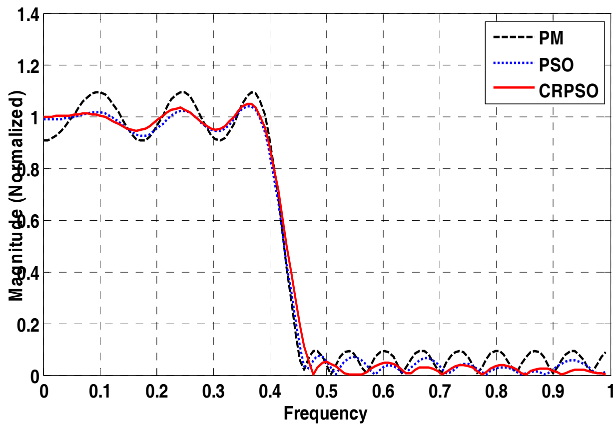

Fig. 4. Normalized Frequency response for the FIR Filter of order 30.

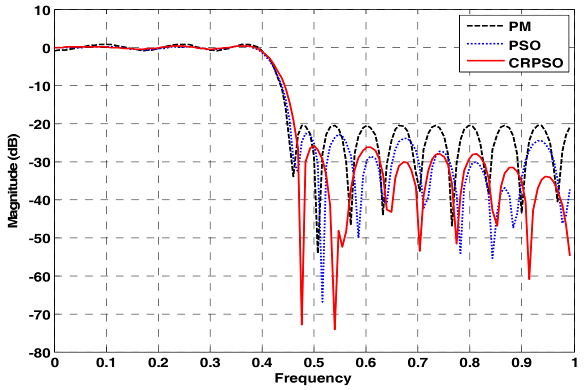

Fig. 5. Gain (dB) Plot of the FIR Filter of order 30.

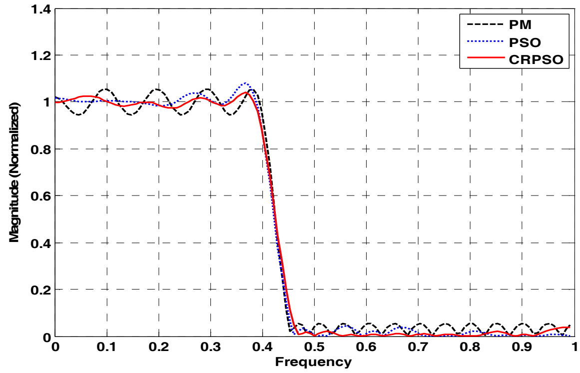

Fig. 6. Normalized Frequency response for the FIR Filter of order 40.

Table 2.

Optimized Filter Coefficients for the FIR filter of order 20

H(N)	PSO	CRPSO
H(1)=H(21)	0.022408286457300	0.019895614917157
H(2)=H(20)	-0.029147712107531	-0.041996103784208
H(3)=H(19)	-0.043973123083511	-0.053810975826974
H(4)=H(18)	0.009526198999545	-0.008213123009929
H(5)=H(17)	0.055415671508159	0.050827622341680
H(6)=H(16)	0.022714060120439	0.032684232836611
H(7)=H(15)	-0.072195797899834	-0.057362493416407
H(8)=H(14)	-0.067827560918525	-0.070214680502808
H(9)=H(13)	0.078603595140571	0.070573334804444
H(10)=H(12)	0.301031346152943	0.296256191283642
H(11)	0.424852775587723	0.424800670522855

Table 3.

Optimized Filter Coefficients for the FIR filter of order 30

H(N)	PSO	CRPSO
H(1)=H(31)	0.022950203459264	0.016137130589114
H(2)=H(30)	-0.005984343078409	-0.005206618657744
H(3)=H(29)	-0.026912021362354	-0.023055331668834
H(4)=H(28)	-0.010674555089066	-0.008813949545769
H(5)=H(27)	0.019226714230719	0.012260667376161
H(6)=H(26)	0.015996855131320	0.022679403163700
H(7)=H(25)	-0.015175152933991	-0.009443431710420
H(8)=H(24)	-0.035661239760689	-0.032005090671930
H(9)=H(23)	0.005824456995417	-0.001343631142177
H(10)=H(22)	0.053371741025761	0.050322645950852
H(11)=H(21)	0.022940616917854	0.031498730994013
H(12)=H(20)	-0.057547352445369	-0.058847892502176
H(13)=H(19)	-0.077361722742027	-0.083894280282535
H(14)=H(18)	0.071928817380659	0.069834919034240
H(15)=H(17)	0.300000534093426	0.307436782689523
H(16)	0.424086365330878	0.424115577461817

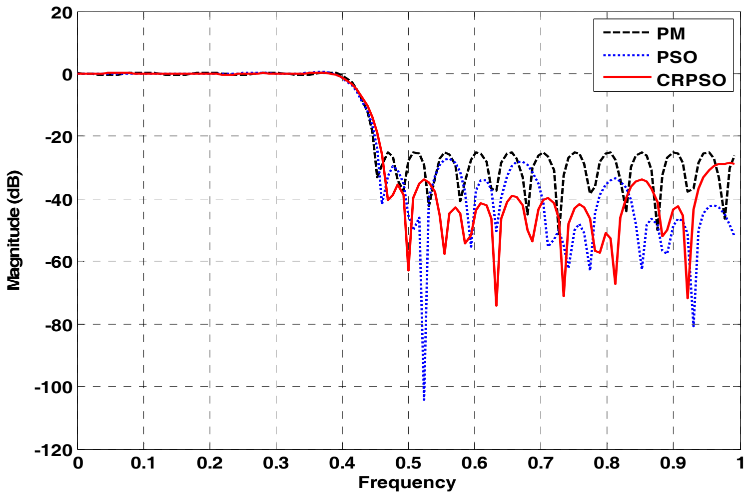

Fig. 7. Gain (dB) Plot of the FIR Filter of order 40.

Table 4.

Optimized Filter Coefficients for the FIR filter of order 40

H(N)	PSO	CRPSO
H(1)=H(41)	0.007344512754673	0.006514733837337
H(2)=H(40)	-0.000759066513506	-0.000793546803820
H(3)=H(39)	-0.012955069139996	-0.009715934911209
H(4)=H(38)	-0.003429993950846	-0.007496804419303
H(5)=H(37)	0.013452816832404	0.005458238309970
H(6)=H(36)	0.013414622028248	0.012172026646061
H(7)=H(35)	-0.005407458851593	-0.003872743850380
H(8)=H(34)	-0.017445105936959	-0.016918320492523
H(9)=H(33)	-0.005831105005441	-0.008166009176146
H(10)=H(32)	0.020098094309249	0.020419995452682
H(11)=H(31)	0.019550305005331	0.016112117294538
H(12)=H(30)	-0.019726721713318	-0.015600387866908
H(13)=H(29)	-0.035204199271079	-0.034254073511217
H(14)=H(28)	0.007183194774913	0.004685162084647
H(15)=H(27)	0.052107169836207	0.049574987558485
H(16)=H(26)	0.026757554231160	0.027207712041898
H(17)=H(25)	-0.066403671009133	-0.061349758604805
H(18)=H(24)	-0.082929921233006	-0.077016973040211
H(19)=H(23)	0.074863736908197	0.068289602273310
H(20)=H(22)	0.311438927744235	0.312051249215373
H(21)	0.424178535533852	0.423649126868468

Table 5.

Comparison summery of the parameters of interest

Filter Order	Maximum Stop-band ripple (dB)			Maximum Stop-band ripple (Normalized)		
	PM	PSO	CRPSO	PM	PSO	CRPSO
20	-15.63	-17.75	-20.43	0.1661	0.1295	0.0952
30	-20.44	-22.74	-25.94	0.0951	0.0729	0.0505
40	-25.1	-27.17	-28.64	.05542	.04378	0.0369

4.2. Comparative effectiveness and convergence profiles of PSO and CRPSO

In order to compare the algorithms in terms of the error convergence speed, Fig. 8 shows the evolution of best solutions obtained when PSO is employed. Fig. 9 shows the evolution of the best solutions obtained when the proposed CRPSO is employed. The convergence graph has been shown for the filter order of 40. A similar plot can be obtained for the FIR filter of orders 20 and 30. From the figures drawn for this filter, it is seen that the CRPSO algorithm is faster than the PSO algorithm for finding the optimum filter. The CRPSO converges to much lower error fitness in lesser number of iterations, as compared to PSO, which yields suboptimal higher values of error fitness. PSO and CRPSO converge to their respective minimum ripple magnitudes in less than 500 iterations. With a view to the above fact, it may finally be inferred that the performance of CRPSO technique is better as compared to PSO. All optimization programs are run in MATLAB 7.5 version on core (TM) 2 duo processor, 3.00 GHz with 2 GB RAM.

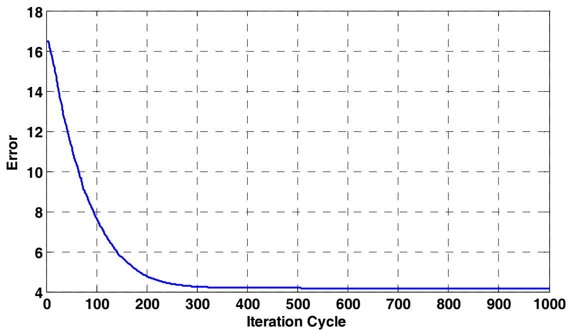

Fig. 8. Convergence Profile for PSO in case of 40th Order Low Pass FIR Filter.

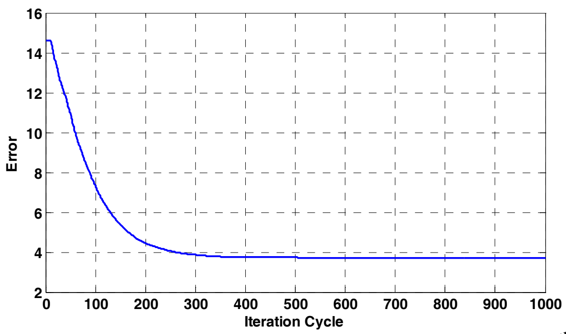

Fig. 9. Convergence Profile for CRPSO in case of 40th Order Low Pass FIR Filter.

5. Conclusions

This paper presents a novel alternative method for designing the linear phase digital low pass FIR filters by using nonlinear stochastic global optimization technique based on Craziness based Particle Swarm Optimization (CRPSO) approach. Filters of orders 20, 30 and 40 have been realized using PM algorithm, conventional PSO as well as the proposed PSO algorithm called CRPSO. Extensive simulation results justify that the proposed algorithm outperforms PSO and PM in the accuracy of the magnitude response of the filter as well as in the convergence speed and is adequate for use in other related design problems.

REFERENCES

1. Litwin, L.: *FIR and IIR digital filters*, IEEE Potentials, pp. 0278-6648, 2000, 28–31.
2. Parks, T.W., Burrus, C.S., *Digital Filter Design*, Wiley, New York, 1987.
3. Parks, T.W., McClellan, J.H.: *Chebyshev approximation for non recursive digital filters with linear phase*, IEEE Trans. Circuits Theory CT-19 (1972), 189–194.
4. McClellan, J.H., Parks, T.W., Rabiner, L.R.: *A computer program for designing optimum FIR linear phase digital filters*, IEEE Trans. Audio Electro acoust., AU-21 (1973) 506–526.
5. Rabiner, L.R.: *Approximate design relationships for low-pass FIR digital filters*, IEEE Trans. Audio Electro acoust., AU-21 (1973) 456–460.
6. Lin, Z.: *An introduction to time-frequency signal analysis*, Sensor Review, vol. 17, pp. 46–53, 1997.
7. Chen, S.: *IIR Model Identification Using Batch-Recursive Adaptive Simulated Annealing Algorithm*, In Proceedings of 6th Annual Chinese Automation and Computer Science Conference, 2000, pp.151–155.
8. Mastorakis, N.E., Gonos, I.F., Swamy, M.N.S.: *Design of Two Dimensional Recursive Filters Using Genetic Algorithms*, IEEE Transaction on Circuits and Systems-I; Fundamental Theory and Applications, 50 (2003) 634–639.
9. Karaboga, N.: *A new design method based on artificial bee colony algorithm for digital IIR filters*. Journal of the Franklin Institute, 346(4), 2009, 328–348.
10. Kennedy, J., Eberhart, R.: *Particle Swarm Optimization*, in Proc. IEEE int. Conf. On Neural Network, 1995.
11. Eberhart, R., Shi, Y.: *Comparison between Genetic Algorithms and Particle Swarm Optimization*, Proc. 7th Ann. Conf. on Evolutionary Computation, San Diego, 2000.
12. Ababneh, J.I., Bataineh, M. H.: *Linear phase FIR filter design using particle swarm optimization and*

genetic algorithms, Digital Signal Processing, 18, 657–668, 2008.

13. Mandal, D., Ghoshal, S.P., and Bhattacharjee, A. K.: *A Novel Particle Swarm Optimization Based Optimal Design of Three-Ring Concentric Circular Antenna Array*, IEEE International Conference on Advances in Computing, Control, and Telecommunication Technologies, 2009, (ACT'09), pp. 385-389, 2009.
14. Ling, S. H., Iu, H. H. C., Leung, F. H. F., and Chan, K.Y.: "*Improved hybrid particle swarm optimized wavelet neural network for modeling the development of fluid dispensing for electronic packaging*," IEEE Trans. Ind. Electron., vol. 55, no. 9, pp. 3447–3460, Sep. 2008.
15. Biswal, B. P., Dash, K., Panigrahi, B. K.: "*Power quality disturbance classification using fuzzy C-means algorithm and adaptive particle swarm optimization*," IEEE Trans. Ind. Electron., vol. 56, no. 1, pp. 212–220, Jan. 2009.
16. Luitel, B., Venayagamoorthy, G.K.: *Differential Evolution Particle Swarm Optimization for Digital Filter Design*, 2008 IEEE Congress on Evolutionary Computation (CEC 2008), PP. 3954-3961, 2008.
17. Herrmann, O., Schussler,W.: *Design of non-recursive digital filters with linear phase*, Electron. Lett., 6 (1970), 329–330.
18. Sarangi, Archana, Mahapatra, R.K., Panigrahi S.K.: *DEPSO and PSO-QI in digital filter design*, Expert Systems with Applications (2011), doi:10.1016/j.eswa.2011.02.140.
19. Mandal, D., Ghoshal S.P., Bhattacharjee, A. K.: *Radiation Pattern Optimization for Concentric Circular Antenna Array With Central Element Feeding Using Craziness Based Particle Swarm Optimization*, International Journal of RF and Microwave Computer-Aided Engineering, John Wiley & Sons, Inc., vol. 20, Issue. 5, pp. 577-586, Sept. 2010.
20. Kar, R., Roy, D., Mandal, D., Ghoshal, S.P.: *FIR Filter Design Using Particle Swarm Optimization with Constriction Factor and Inertia Weight Approach*, Second International Conference on Advances in Computer Engineering – ACE 2011, 25 - 26 Aug 2011 at Trivandrum, Kerala, INDIA (accepted)