

# OPERATION PLANNING OF WIND FARMS WITH PUMPED STORAGE PLANTS BASED ON TYPE-2 FUZZY MODELING OF UNCERTAINTIES

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**Abstract:** *The operation planning problem encounters several uncertainties in terms of the power system's parameters such as load, operating reserve and wind power generation. The modeling of those uncertainties is an important issue in power system operation. The system operators can implement different approaches to manage these uncertainties. These approaches can be categorized into two main groups; mathematical and physical methods. In mathematical methods, different approaches can be used to model the uncertainties of parameters in the formulation of power system problems. A fuzzy based modeling approach can be implemented to develop the new formulation of power system problems under an uncertain environment. On the other hand, in physical methods, energy storage devices can be presented to manage the unpredictable behavior of some parameters of power system. Pumped storage plant is the main energy storage devices that have extensive attraction in system operators.*

*In this paper, the type-2 fuzzy membership function (MF) is implemented to model the uncertainty of wind power generation and the type-1 fuzzy MF is used to model the other parameters in weekly unit commitment (UC) problem. The proposed approach is applied to two different test systems which have conventional generating units, wind farms and pumped storage plants to consider the results of fuzzy modeling of uncertainties.*

**Keywords:** *Unit commitment, type-2 fuzzy sets, wind power uncertainty, pumped storage plant*

## 1. INTRODUCTION

Wind energy is becoming the most important component of renewable energy in the world and is being paid more attention by governments because of its economic and social benefits. Integrating wind energy in the electricity industry and market poses significant challenges to different power system operation problems. The unit commitment (UC) problem becomes more complicated in the wind-thermal coordination scheduling task imposed by considering additional reserve requirements. Because of the relationship between the system reserve requirements and the total actual wind power generation, both of these should be considered at the same time, and not separately. These complex conditions make it very difficult to coordinate the wind-thermal generations to achieve optimal utilization of the wind energy sources [1]. It has been reported that the stochastic models have better performance than the deterministic model under uncertainty in some of the parameters [2], [3]. The

application of fuzzy logic in the UC problem has been demonstrated in [2], [4] and [5].

The general framework of fuzzy reasoning allows handling much of this uncertainty. The type-1 fuzzy sets represent uncertainty using numbers in a range of [0, 1] referred to as the degree of membership. When something is uncertain, like a measurement, it is difficult to determine its exact value, and of course type-1 fuzzy sets make more sense than using crisp sets. However, it is not reasonable to use an accurate membership function for something uncertain, so in this case another type of fuzzy sets is needed to handle these uncertainties, which is called type-2 fuzzy sets. This type of fuzzy sets was introduced by Zadeh [6], as an extension of type-1 fuzzy sets. Therefore, the uncertainty in a system can be modeled in a better way by employing a type-2 fuzzy set which offers better capabilities to cope with linguistic uncertainties by modeling vagueness and unreliability of information [7-9]. The advantage of using type-2 fuzzy sets to deal with uncertain information is recently presented in some research works. The implementation of a type-2 Fuzzy Logic System (FLS) is presented in [10]; while in others, it is explained how type-2 fuzzy sets provide a tool to model and minimize the effects of uncertainties in the rule-base FLSs [11]. The theory and properties of type-2 fuzzy sets are presented in [12-17].

One of the most important strategies for increasing profits of each utility is integrating the wind power resources with limited energy resources such as pumped storage plants. A pumped storage plant can be used to provide added value to a wind farm that is taking part in the market in comparison with separate participation of them. The possibility of storing energy in pumped storage plants can significantly reduce the risk of self-scheduling for wind power producers in the market. Pumped storage units can be used to store the excess energy from wind power and provide the reserve and flexibility needed in systems with large amounts of wind power. Several studies have already addressed the value of storage in power systems with a large amount of wind power [18-22]. All of these studies found that stored energy reduces the system operating cost and makes possible the integration of higher penetration of wind power. Other studies have been tried to develop a decision approach to set different objective functions such as profit

maximization [23], carbon emission reduction [24] curtailment reduction [25]. Pumped storage would also benefit the system by balancing wind power in a market [3] or in an isolated power system [26].

This paper extends UC problem by introducing additional constraints to represent the wind farms generation uncertainties into the problem formulation with pumped storage plants. The main contributions of this work are as follows:

1. A new fuzzy unit commitment method is presented which integrates wind power generation and pumped storage plants,
2. Uncertainty in parameters is simulated by fuzzy sets; especially interval type-2 fuzzy set is used to model wind generation uncertainty,
3. The results of sensitivity analysis of interval type-2 fuzzy modeling are presented and compared.

In next section, the objective function and constraints of UC problem are presented. In this section, the wind turbine and pumped storage models are firstly discussed to present the UC formulation. The uncertain parameters are represented based on application of type-1 and type-2 fuzzy sets in section 3. The approach of converting the fuzzy optimization to crisp optimization for fuzzy UC is presented in Section 4. The General Algebraic Modeling System (GAMS) has been used to solve this mixed integer nonlinear problem using BARON (Branch And Reduced Optimization Navigator) optimization program. And in section 5, two test systems which have 6 and 26 conventional generating units are used to demonstrate this optimization problem advantages based on the proposed method developed. Both test systems have two wind farms and two pumped storage plants. Summary and conclusion are presented in Section 6.

## 2. UC FORMULATION

### 2.1. Wind Farm Model

The generated power varies with the wind speed at the wind farm (WF) site. The power output of a wind turbine can be determined from its power curve, which is a plot of output power against wind speed. A turbine is designed to start generating at the cut-in wind speed ( $V_{ci}$ ) and is shut down for safety reasons at the cut-out wind speed ( $V_{co}$ ). Rated power  $P_r$  is generated, when the wind speed is between the rated wind speed ( $V_r$ ) and the cut-out wind speed. There is a nonlinear relationship between the power output and the wind speed when the wind speed lies within the cut-in and the rated wind speed as shown in Figure 1.

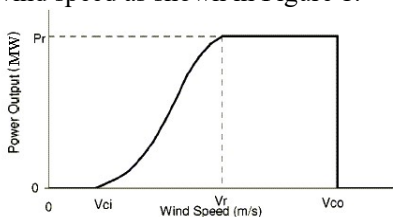


Figure 1. Power Curve of a Wind Turbine

Therefore, the wind power generated corresponding to a given wind speed can be obtained from:

$$W_{av}(w,t) = P_r * \begin{cases} A + B * WS(w,t) + C * WS(w,t)^2 & V_{ci} \leq WS(w,t) < V_r \\ 1 & V_r \leq WS(w,t) \leq V_{co} \\ 0 & \text{Otherwise} \end{cases} \quad (1)$$

Where the constants A, B, and C are presented in [27]. The application of the common wind power generation model is illustrated in this paper by applying it to a wind turbine rated at 2 MW, and with cut-in, rated, and cut-out wind speeds of 3.5, 12.5, and 25 m/s, respectively.

### 2.2. Pumped-Storage Model

The pumped-storage plant (PS) is composed of an upper and lower reservoirs. Typically, a reversible pump-turbine makes possible the storing of energy in off-peak hours that it can be sold during peak hours, provided that the operation is economically profitable. Thus, the pump-turbine will work as a turbine when water is released from the upper reservoir to the lower one, injecting its production to the network. Likewise, when pumping is taking place, the energy is consumed to store water in the upper reservoir, which will be available later on for hydroelectric generation. The variables associated to the pumped-storage plant in the model are considered in terms of energy. Thus, in each period, the state of the upper and lower reservoirs will be determined by the energy stored in them at the end of the period. Likewise, the volume capacity of both reservoirs will be expressed as a maximum and minimum energy level that can be stored in the reservoirs [28].

The profit of pumped storage plant can be divided in two parts. At first, this plant can sell energy to the market based on forecasted energy price and next, this plant can participate in operating reserve market based on maximum capacity of pumped storage plant or when it is not in the generating mode. But, in pumping mode, these plants have to buy energy from the energy market and in this situation; the plant can reduced the load of pumping mode then can be committed to operating reserve market. Thus, there are strong incentives for pumped storage plants in a competitive electricity market. The energy stored in each lower and upper reservoirs of pumped storage plant has an upper and a lower capacity limits which are:

$$Eu_{\min}(t) \leq Eu(t) \leq Eu_{\max}(t) \quad (2)$$

$$El_{\min}(t) \leq El(t) \leq El_{\max}(t) \quad (3)$$

In this paper, the pumped storage plant can be participated in reserve power market when it work in generating mode of operation and capability of reducing the load of pumping mode is not considered.

### 2.3. UC Formulation

The main objective of a UC problem is to maximize the total profit of its generating units in the scheduled horizon. While, the operation is constrained by a number of system and generating units' constraints, beside the uncertainty that exist in some of the modeling parameters. The time horizon of this problem is one week, with hourly intervals. The objective function of UC problem is defined as follows:

$$\text{Max } J = TR - TC \quad (4)$$

Where:

$$\begin{aligned} TR = & \sum_{t=1}^T \sum_{g=1}^{N_G} \{P_{GD}(g,t) \cdot U(g,t)\} \cdot EP(t) \\ & + \sum_{t=1}^T \sum_{w=1}^{N_W} \{P_W(w,t) \cdot V(w,t)\} \cdot EP(t) \\ & + \sum_{t=1}^T \sum_{s=1}^{N_S} \{PS_g(s,t) \cdot M(s,t)\} \cdot EP(t) \\ & + \sum_{t=1}^T \left\{ \sum_{g=1}^{N_G} \{P_{GR}(g,t) \cdot U(g,t)\} - RESW \cdot \sum_{w=1}^{N_W} \{P_W(w,t) \cdot V(w,t)\} \right\} \cdot RP(t) \\ & + \sum_{t=1}^T \sum_{s=1}^{N_S} \{PS_{g,\max}(s) - PS_g(s,t) \cdot M(s,t)\} \cdot PSRES \cdot RP(t) \end{aligned} \quad (5)$$

$$\begin{aligned} TC = & \sum_{t=1}^T \sum_{g=1}^{N_G} F(P_{GD}(g,t)) \cdot U(g,t) \\ & + \sum_{t=1}^T \sum_{g=1}^{N_G} SU(g) \cdot U(g,t) \cdot (1 - U(g,t-1)) \\ & + \sum_{t=1}^T \sum_{g=1}^{N_G} \{(P_{GD}(g,t) + P_{GR}(g,t)) \cdot OMVCT(g)\} \cdot U(g,t) \\ & + \sum_{t=1}^T \sum_{s=1}^{N_S} \{PS_p(s,t) \cdot (1 - M(s,t))\} \cdot EP(t) \\ & + \sum_{t=1}^T \sum_{w=1}^{N_W} \{P_W(w,t) \cdot OMVCW(w)\} \cdot V(w,t) \end{aligned} \quad (6)$$

Where:

$$F(P_{GD}(g,t)) = a_g + b_g \cdot P_{GD}(g,t) + c_g \cdot P_{GD}(g,t)^2 \quad (7)$$

This objective function is subjected to many constraints; including: the forecasted demand, the reserve power requirement, the generating units' constraints, and the wind power and pumped storage generation.

The demand constraint is arranged by an equality function which is defined as a fuzzy equality. To satisfy the forecasted demand, the following fuzzy equation should be valid:

$$\begin{aligned} \sum_{g=1}^{N_G} P_{GD}(g,t) \cdot U(g,t) + \sum_{w=1}^{N_W} P_W(w,t) \cdot V(w,t) \\ + \sum_{s=1}^{N_S} PS_g(s,t) \cdot M(s,t) - \sum_{s=1}^{N_S} PS_p(s,t) \cdot (1 - M(s,t)) \cong P_d(t) \end{aligned} \quad t = 1, 2, \dots, T \quad (8)$$

The operating reserve requirement has two parts; first part is a percentage of the forecasted demand (e.g. 5% of demand) and the second part is a surplus reserve which is chosen to compensate the mismatch between the forecasted wind power generation and its actual value. It is assumed that the second part of reserve is determined using a percentage of total wind power availability (RESW) [1]. Therefore, system operator

must provide the more reserve power because of uncertainty in wind power generation. In this paper, a certain percentage of available wind power is assumed to build the extra reserve power. The reserve requirement (both parts) could be provided through the conventional units and excess capacity of pumped storage plants in generating mode based on the forecasted reserve power price. The reserve power requirement which is defined as a fuzzy inequality should be satisfied as follows:

$$\begin{aligned} \sum_{g=1}^{N_G} P_{GD}(g,t) \cdot U(g,t) + PSRES \cdot \sum_{s=1}^{N_S} \{PS_{g,\max}(s) - PS_g(s,t) \cdot M(s,t)\} \\ - RESW \cdot \sum_{w=1}^{N_W} P_W(w,t) \cdot V(w,t) \geq P_r(t) \end{aligned} \quad t = 1, 2, \dots, T \quad (9)$$

The wind power generation and the available wind power should satisfy the fuzzy equality relation. The fuzzy equality is expressed as a type-2 fuzzy membership function ( $\tilde{W}_{av}$ ) which is explained in next section. This relation can be shown as follows:

$$P_W(w,t) \cong \tilde{W}_{av}(w,t) \quad t = 1, 2, \dots, T \quad (10)$$

The maximum and minimum generation limits of the conventional generating unit should be satisfied as follows:

$$P_{Gg,\min} \leq P_{GD}(g,t) + P_{GR}(g,t) \leq P_{Gg,\max} \quad t = 1, 2, \dots, T \quad (11)$$

Other conventional unit constraints such as ramp up and ramp down limits and also, up-time and down-time limits in each period can be obtained by:

$$-RD_g \leq P_{GD}(g,t) - P_{GD}(g,t-1) \leq RU_g \quad t = 1, 2, \dots, T \quad (12)$$

$$U(g,t+1) = \begin{cases} 1 & \text{if } U(g,t)=1 \text{ and } t_g^{up} < UT_g \\ 0 & \text{if } U(g,t)=0 \text{ and } t_g^{down} < DT_g \\ 0 \text{ or } 1 & \text{otherwise} \end{cases} \quad (13)$$

Consider a pumped storage unit having an efficiency of pumping ( $\eta$ ) with an initial energy stored in the lower and upper reservoirs. Also, assume that within a time period of study horizon, the stored energy in both reservoirs is the same as initial states. The maximum and minimum energy storing in upper and lower reservoirs of pumped storage plant should be calculated and satisfied as follows:

$$\begin{aligned} Eu_{\min}(s) \leq Eu(s,t) = Eu(s,t-1) - PS_g(s,t) \times M(s,t) \\ + \eta(s) \times [PS_p(s,t) \times (1 - M(s,t))] \leq Eu_{\max}(s) \end{aligned} \quad (14)$$

$$\begin{aligned} El_{\min}(s) \leq El(s,t) = El(s,t-1) + PS_g(s,t) \times M(s,t) \\ - \eta(s) \times [PS_p(s,t) \times (1 - M(s,t))] \leq El_{\max}(s) \end{aligned} \quad (15)$$

## 3. FUZZY OPTIMIZATION

### 3.1. Fuzzy Concepts and Type 2 Fuzzy Sets

The concept of type-2 fuzzy set is an extension of the type-1 set. A type-2 fuzzy set expresses the non-deterministic truth degree with imprecision and

uncertainty for an element that belongs to a set. Then, at a specific value  $x'$ , the MF value  $u'$ , takes on different values which not all be weighted the same, so an amplitude distribution can be assigned to all of those points. Doing this for all  $x \in X$ , a three-dimensional MF—a type-2 membership function—that characterizes a type-2 fuzzy set [15] is created. Hence, a type-2 membership grade can be any subset of interval  $[0, 1]$ , as a primary membership. There is a secondary membership that may correspond to each parameter of the primary membership (can also be in  $[0, 1]$ ) and presents the uncertainty in the primary membership. This uncertainty is represented by a region called footprint of uncertainty (FOU).

An interval type-2 fuzzy set is one in which the membership grade of every domain point is a crisp set whose domain is some interval contained in interval  $[0, 1]$ . Interval type-2 fuzzy set is an especial case of general type-2 fuzzy sets. The membership grade of a type-2 interval fuzzy set is an interval set, with a unity value for each secondary grade in that set [29].

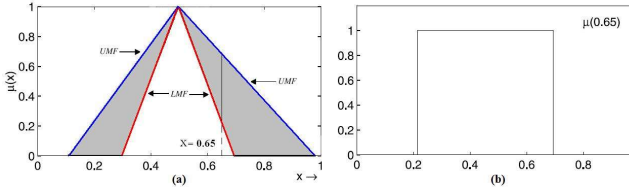


Figure 2. (a) Interval Type-2 Fuzzy Set (b) Secondary MF at  $x=0.65$

Figure 2 shows an interval type-2 fuzzy set with triangular footprint of uncertainty. The primary membership at  $x=0.65$  is also shown. The secondary MF at  $x=0.65$  is shown in Figure 2(b) and it equals 1, i.e., the secondary MF is an interval type-1 fuzzy set. The uniform shading for the FOU represents the entire interval type-2 fuzzy set and it can be described by area which is bounded between an upper MF (UMF)  $\bar{\mu}_{\tilde{A}}(x)$  and a lower MF (LMF)  $\underline{\mu}_{\tilde{A}}(x)$  (Fig. 2(a)).

On the other hand, type-2 fuzzy sets are very useful in circumstances where it is difficult to determine an exact and certainty of the measurement uncertainties. The symmetrical interval type-2 fuzzy sets, whose lower MF and upper MF are characterized by the width of MF, are implemented in this paper.

### 3.2. Fuzzy Modeling of Parameters

To obtain an optimal unit commitment under the uncertainty environment: total profit, forecasted load, forecasted reserve power and wind power generation constraints are all expressed in fuzzy equality or inequality function. Also, these fuzzy equations are combined with other crisp constraints including; limits on capacity of thermal units, wind farms' outputs constraints and pumped storage energy constraints. Now, these MFs will be defined for each equation as follows.

#### 3.2.1. Load balance membership function

The membership function of the fuzzy equality ( $\cong$ ) in (8) can be described as (16). Figure 3a shows the MF of forecasted demand equality ( $P_d$ ) which is defined for each time period of demand ( $P_d(t)$ ). In this study, the maximum range of variation of the predicted demand ( $\Delta P_d$ ) is assumed to be equal to 5%.

$$\mu_{P_d}(x) = \begin{cases} 1 & x = P_d \\ \frac{x - P_d + \Delta P_d}{\Delta P_d} & P_d - \Delta P_d \leq x \leq P_d \\ \frac{P_d + \Delta P_d - x}{\Delta P_d} & P_d \leq x \leq P_d + \Delta P_d \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

#### 3.2.2. Reserve power generation membership function

The reserve power generation constraint can be described through a fuzzy inequality relation ( $\lesssim$ ), i.e., the total reserve power generation contribution at time period  $t$  could roughly be less than or equal to the forecasted reserve power generation at that time. So, MF of reserve power ( $P_R$ ) is defined for each time period of reserve ( $P_R(t)$ ). The MF of the fuzzy inequality of reserve power generation contribution in (9) is described by:

$$\mu_{P_R}(x) = \begin{cases} 1 & x \leq P_R \\ \frac{P_R + \Delta P_R - x}{\Delta P_R} & P_R \leq x \leq P_R + \Delta P_R \\ 0 & x \geq P_R + \Delta P_R \end{cases} \quad (17)$$

Where  $P_R$  is the predicted reserve contribution at time period  $t$ , and  $P_R + \Delta P_R$  is the maximum reserve power generation contribution. In this study, the predicted reserve power generation is assumed to be 5% of the forecasted load demand and also  $\Delta P_R$  is assumed to be 5% of this reserve. Figure 3b shows the MF of reserve power inequality.

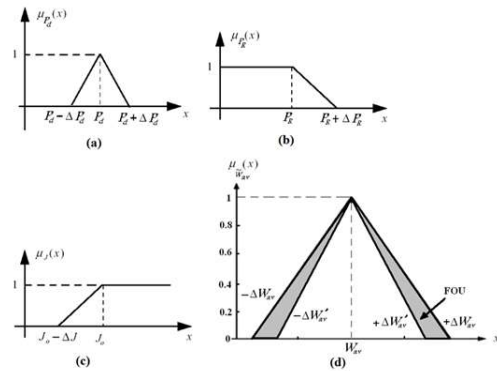


Figure 3. Fuzzy MF of (a) Forecasted Load Equality, (b) Reserve Power Inequality, (c) Total Profit and (d) Type-2 MF of Wind Power Generation Equality

### 3.2.3. Available wind power membership function

The wind power prediction error is obtained, employing the wind speed prediction error, and the non-linear wind power characteristic curve. Thus, the available wind power constraint can be described as a fuzzy equality relation ( $\cong$ ). Figure 3d shows the MF of wind power generation equality based on interval type-2 fuzzy set. The upper and lower MFs have been shown the maximum range of uncertainty in the wind power generation ( $\Delta W_{av}$  and  $\Delta W'_{av}$ , respectively).

The MF of the type-2 fuzzy equality of available wind power for upper and lower bound of footprint of uncertainty (FOU) are respectively described by:

$$\bar{\mu}_{\tilde{w}_{av}}(x) = \begin{cases} 1 & x = W_{av} \\ \frac{x - W_{av} + \Delta W_{av}}{\Delta W_{av}} & W_{av} - \Delta W_{av} \leq x \leq W_{av} \\ \frac{W_{av} + \Delta W_{av} - x}{\Delta W_{av}} & W_{av} \leq x \leq W_{av} + \Delta W_{av} \\ 0 & \text{otherwise} \end{cases} \quad (18)$$

$$\underline{\mu}_{\tilde{w}_{av}}(x) = \begin{cases} 1 & x = W'_{av} \\ \frac{x - W'_{av} + \Delta W'_{av}}{\Delta W'_{av}} & W'_{av} - \Delta W'_{av} \leq x \leq W'_{av} \\ \frac{W'_{av} + \Delta W'_{av} - x}{\Delta W'_{av}} & W'_{av} \leq x \leq W'_{av} + \Delta W'_{av} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

### 3.2.4. Objective equation membership function

The objective function equation can be described as a fuzzy inequality relation ( $\gtrsim$ ). As mentioned above, the total profit of UC problem should be essentially greater than or equal to some aspiration level  $J_0$ :

$$\text{Max } J \gtrsim J_0 \quad (20)$$

The MF for the fuzzy inequality in (20) is assumed to be as follows (Fig. 3c):

$$\mu_J(x) = \begin{cases} 1 & x \geq J_0 \\ \frac{x - J_0 + \Delta J}{\Delta J} & J_0 - \Delta J \leq x \leq J_0 \\ 0 & x \leq J_0 - \Delta J \end{cases} \quad (21)$$

The aspiration level  $J_0$  represents the expected total profit. A generation scheduling output with the profit less than the expected total profit ( $J_0$ ) is indicated by membership value less than one. The value  $\Delta J$  can be determined as a certain percentage of  $J_0$  (in this study, it is assumed to be 90%). The overall scheduling problem with fuzzy objective and constraints can thus be formulated through the satisfaction of (20) subject to (16) to (19), with other crisp constraints of UC problem.

## 4. SOLUTION METHODOLOGY

The key step of fuzzy optimization is to convert the fuzzy problem to a crisp one. Since all the fuzzy objective and constraints are desired to be satisfied simultaneously. The problem is to maximize the degree to which all the constraints (including the objective function constraint) are satisfied. The decision variable  $z$  is defined as the minimum degree of satisfaction among all fuzzy constraints as follows:

$$\max_{z \in [0,1]} z = \max_{z \in [0,1]} \left\{ \min \{ \mu_J, \mu_{P_d(t)}, \mu_{P_r(t)}, \mu_{\tilde{w}_{av}(w,t)} \} \right\} \quad t = 1, 2, \dots, T \quad w = 1, 2, \dots, N_W \quad (22)$$

Now, based on interval type-2 MF of available wind power, this equation is changed to:

$$\max_{z \in [0,1]} z = \max_{z \in [0,1]} \left\{ \min \{ \mu_J, \mu_{P_d(t)}, \mu_{P_r(t)}, [\underline{\mu}_{\tilde{w}_{av}(w,t)}, \bar{\mu}_{\tilde{w}_{av}(w,t)}] \} \right\} \quad t = 1, 2, \dots, T \quad w = 1, 2, \dots, N_W \quad (23)$$

Then,

$$\max_{z \in [0,1]} z = \max_{z \in [0,1]} \left\{ \frac{1}{2} \left[ \min \{ \mu_J, \mu_{P_d(t)}, \mu_{P_r(t)}, \bar{\mu}_{\tilde{w}_{av}(w,t)} \} + \min \{ \mu_J, \mu_{P_d(t)}, \mu_{P_r(t)}, \underline{\mu}_{\tilde{w}_{av}(w,t)} \} \right] \right\} \quad t = 1, 2, \dots, T \quad w = 1, 2, \dots, N_W \quad (24)$$

Figure 4 shows the concept of this relation especially for interval type-2 fuzzy set of wind generation membership variable.

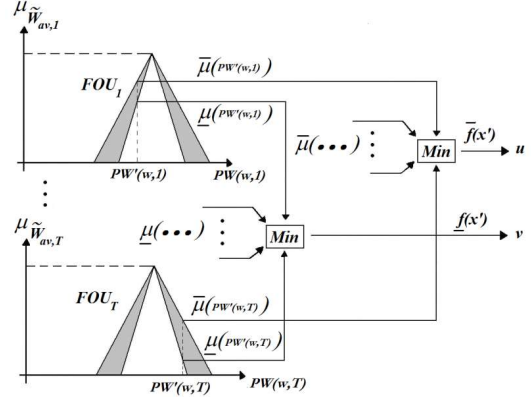


Figure 4. Operation on Interval Type-2 Fuzzy MF of Available Wind Power

In this figure, the membership variable of other type-1 fuzzy sets is not shown. The equation (24) can be rewritten as follows:

$$\text{Max } z = \frac{u + v}{2} \quad (25)$$

Subject to:

$$\begin{aligned} u &\leq \mu_J & v &\leq \mu_J \\ u &\leq \mu_{P_d(t)} & v &\leq \mu_{P_d(t)} & t &= 1, 2, \dots, T \\ u &\leq \mu_{P_r(t)} & v &\leq \mu_{P_r(t)} & t &= 1, 2, \dots, T \\ u &\leq \bar{\mu}_{\tilde{w}_{av}(w,t)} & t &= 1, 2, \dots, T & w &= 1, 2, \dots, N_W \\ v &\leq \underline{\mu}_{\tilde{w}_{av}(w,t)} & t &= 1, 2, \dots, T & w &= 1, 2, \dots, N_W \\ 0 &\leq z & & & & \leq 1 \end{aligned}$$

and all other crisp constraints.

Substituting the MFs (16) to (19) into (25), the fuzzy optimization problem can be converted to the following crisp optimization problem:

$$\text{Max } z = \frac{u + v}{2} \quad (26)$$



Subject to:

$$(q-1) \cdot \Delta J + J_0 - J \leq 0$$

$$(q-1) \cdot \Delta P_d + \sum_{g=1}^{N_g} P_{cd}(g,t) \cdot U(g,t) + \sum_{w=1}^{N_w} P_w(w,t) \cdot V(w,t) + \sum_{s=1}^{N_s} PS_g(s,t) \cdot M(s,t) - \sum_{s=1}^{N_s} PS_p(s,t) \cdot (1-M(s,t)) - P_d(t) \leq 0$$

$$(q-1) \cdot \Delta P_d - \sum_{g=1}^{N_g} P_{cd}(g,t) \cdot U(g,t) - \sum_{w=1}^{N_w} P_w(w,t) \cdot V(w,t) - \sum_{s=1}^{N_s} PS_g(s,t) \cdot M(s,t) + \sum_{s=1}^{N_s} PS_p(s,t) \cdot (1-M(s,t)) - P_d(t) \leq 0$$

$$(q-1) \cdot \Delta P_r + \sum_{g=1}^{N_g} P_{gr}(g,t) \cdot U(g,t) + PSRES \cdot \sum_{s=1}^{N_s} \{PS_{g,max}(s) - PS_g(s,t) \cdot M(s,t)\} - RESW * \sum_{w=1}^{N_w} P_w(w,t) \cdot V(w,t) - P_r(t) \leq 0$$

$$(u-1) \cdot \Delta W_{on}(w,t) - W_{on}(w,t) + P_w(w,t) \leq 0$$

$$(u-1) \cdot \Delta W'_{on}(w,t) + W'_{on}(w,t) - P_w(w,t) \leq 0$$

$$(v-1) \cdot \Delta W'_{off}(w,t) - W'_{off}(w,t) + P_w(w,t) \leq 0$$

$$(v-1) \cdot \Delta W_{off}(w,t) + W_{off}(w,t) - P_w(w,t) \leq 0$$

$$u \leq q$$

$$v \leq q$$

$$P_{G,min} \leq P_{cd}(g,t) + P_{gr}(g,t) \leq P_{G,max}$$

$$Eu_{min}(s) \leq Eu(s,t) = Eu(s,t-1) - PS_g(s,t) \cdot M(s,t) + \eta(s) \cdot [PS_p(s,t) \cdot (1-M(s,t))] \leq Eu_{max}(s)$$

$$El_{min}(s) \leq El(s,t) = El(s,t-1) + PS_g(s,t) \cdot M(s,t) - \eta(s) \cdot [PS_p(s,t) \cdot (1-M(s,t))] \leq El_{max}(s)$$

$$-RD_g \leq P_{cd}(g,t) - P_{cd}(g,t-1) \leq RU_g$$

$$U(g,t+1) = \begin{cases} 1 & \text{if } U(g,t)=1 \text{ and } t_g^{\text{up}} < UT_g \\ 0 & \text{if } U(g,t)=0 \text{ and } t_g^{\text{down}} < DT_g \\ 0 \text{ or } 1 & \text{otherwise} \end{cases}$$

Note that all the other crisp constraints still have to be satisfied and  $J$  must be substituted by equation (4). In the membership problem, the optimal membership variable  $z$  tends to decrease as the profit and other constraints' violations become larger. The membership variable  $z$  may become less than one, implying that not all normal constraints can be satisfied.

## 5. NUMERICAL TESTING RESULTS

To examine the merits of the proposed method, two test systems are simulated in this section. The impact of wind power uncertainty is analyzed, first; by employing interval type-2 fuzzy sets and then the related results are compared against each other. This model is developed in GAMS [30] environment. The GAMS solves this optimization problem using the BARON optimization program based on the Mixed Integer Non Linear Programming (MINLP) method.

The decision variable ( $z$ ), representing the degree of satisfaction, can be used as a criterion for operation planning. However, it can be combined with the total profit (obtained by different values of expectation profit  $J_0$ ), in UC study. Based on these two variables, a new criterion index (CI) is defined as follows:

$$CI = z \cdot \left( \frac{TP_i - \text{Min}\{TP_i\}}{\text{Max}\{TP_i\} - \text{Min}\{TP_i\}} \right) \quad (27)$$

### 5.1. Test System 1(6C+2W+2PS)

This test system has six conventional generating units, two wind farms and two pumped storage plants (briefly: 6C+2W+2PS). The input data of this test system including two wind farms (wind1 and wind2) are given in Table 1. Each wind farms has 40 wind turbine units with 2 MW capacities. The annual peak load is predicted to be 300 MW for this study. The forecasted load at each time interval of the study period is shown in Figure 5.

Table 1: Generator Characteristics and Cost Function Coefficients

Parameters	Unit 1	Unit 2	Unit 3	Unit 4	Unit 5	Unit 6	Wind 1	Wind 2
$P_{G,max}$ (MW)	50	60	100	120	100	60	80	80
$P_{G,min}$ (MW)	10	10	10	10	10	10	0	0
Variable O&M Cost (\$/MWh)	0.9	0.9	0.8	0.8	0.8	0.9	3	2
a (\$/hr)	500	650	700	450	500	600	-	-
b (\$/MWh)	25	26.5	18	16	15	27.5	-	-
c (\$/MW <sup>2</sup> h)	0.01	0.012	0.004	0.006	0.004	0.01	-	-
Minimum Up Time (Hour)	2	2	3	4	3	2	-	-
Minimum Down Time (Hour)	2	2	3	4	3	2	-	-
Ramp Up Rate (MW/h)	10	10	20	20	20	10	-	-
Ramp Down Rate (MW/h)	10	10	20	20	20	10	-	-

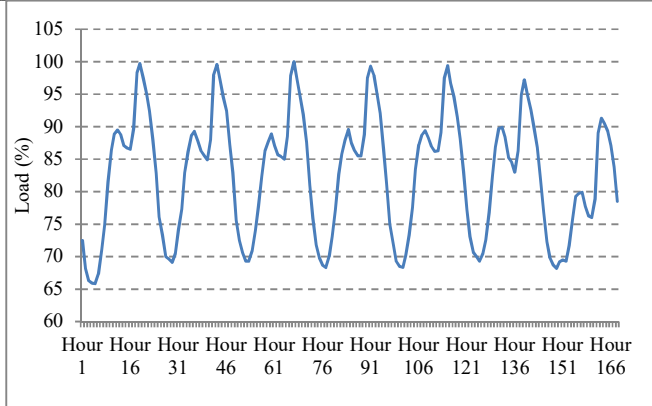


Figure 5. Forecasted Hourly Load

The variation of available wind power generations of these two wind farms during the study time are shown in Figure 6. The forecasted market prices for energy and reserve power are shown in Figure 7. In this study, the RESW is assumed to be 10% of the total available wind power of two wind farms.

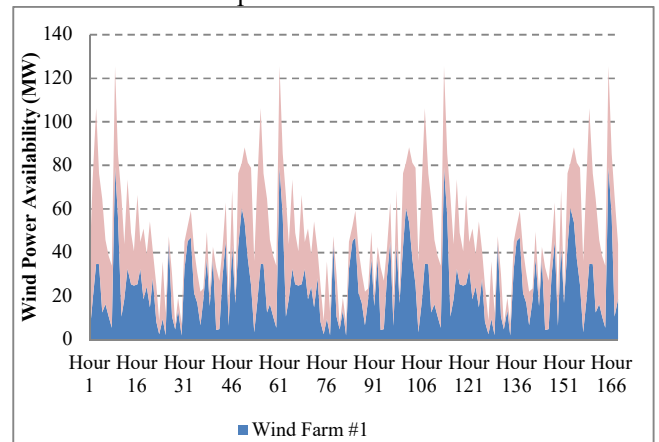


Figure 6. Available Wind Power Generation of Wind Farms

Both pumped storage plants have the same efficiency of 80% and the maximum capacity of generating and pumping of both plants are 90 and 80 MW, respectively. The maximum and minimum

capacity of energy storage in upper dam is assumed 1250 and 450 MWh and for lower dam are 800 and 0 MWh, respectively. The running cost of pumped storage plants are ignored in both generating and pumping modes. The reserve contribution of pumped storage plant (PSRES) is assumed to be 10% in this study.

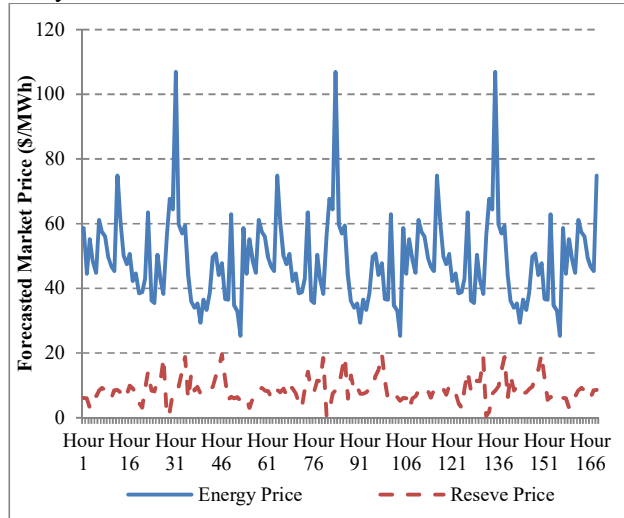


Figure 7. Forecasted Energy and Reserve Power Market Prices

Table 2 shows these results of UC problem using the proposed approach (based on type-2 MF) for the available wind power generation based on different value of aspiration level ( $J_0$ ). In this table, the value of objective function ( $z$ ) and MF value of other parameters have been shown. Based on criterion index, the Run #8 has the best result of proposed method implemented on UC problem.

Table 2: Results of 14 runs of Type-2 Fuzzy Optimization Solution of Test Case #1 ( $\Delta W_{av} = \%5$  and  $\Delta W'_{av} = \%2$ )

Run	$J_0$ (\$/week)	$z$	Total Profit (\$/week)	Criterion Index (CI)	Minimum Value of MF of Parameters in All Periods			
					$\mu_{P_d}$	$\mu_{P_r}$	$\mu_{w_{av}}(1)$	$\mu_{w_{av}}(2)$
#1	685,640	1	716,636	0.000	0.999	0.176	0.999	0.999
#2	785,640	0.997	812,659	0.163	0.999	0.203	0.999	0.999
#3	992,640	0.913	944,578	0.350	0.996	0.194	1.001	1.001
#4	1,005,640	0.968	1,005,922	0.472	0.999	0	1.000	1.000
#5	1,075,640	0.966	1,071,791	0.578	1.001	0	1.000	1.000
#6	1,212,640	0.808	1,032,483	0.429	0.990	0	0.996	0.996
#7	1,412,640	0.732	1,225,188	0.473	0.986	0	1.002	1.002
#8	1,695,640	0.72	1,297,766	0.703	0.985	0.097	1.005	1.005
#9	1,795,640	0.675	1,299,950	0.661	1.016	0.219	1.006	1.006
#10	1,835,640	0.654	1,293,468	0.633	0.982	0	1.006	1.006
#11	1,885,640	0.617	1,265,381	0.568	0.980	0	1.007	1.007
#12	2,015,640	0.593	1,307,803	0.587	0.979	0	1.008	1.008
#13	2,215,640	0.532	1,311,631	0.532	1.023	0.225	1.008	1.009
#14	2,315,640	0.488	1,279,729	0.460	0.974	0	1.008	1.010
#15	2,615,640	0.43	1,302,098	0.423	0.979	0	1.008	1.011
#16	3,215,640	0.32	1,315,572	0.301	0.965	0	0.986	0.986

The UC results of each conventional unit supplying forecasted load during 168 hours (Run #8) is presented in Figure 8 and the pumped storage output in both

pumping and generating modes is shown in Figure 9. The capacity of lower and upper reservoirs of each pumped storage plants are shown in Figure 10. Also, the results of total generation of all units (conventional units, wind farms and generating mode of pumped storage plants) and all demand (native load and pumping mode of pumped storage plants) are shown in Figure 11 and Figure 12 based on Run #8 of Table 2, respectively.

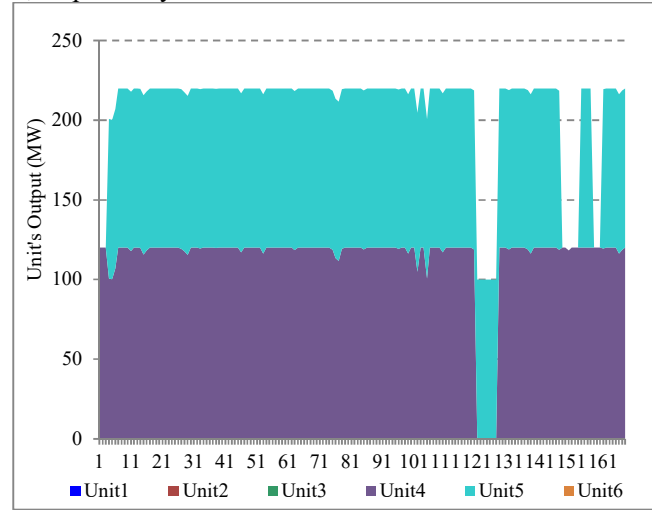


Figure 8. The UC Results of Each Conventional Units Supplying Forecasted Load (Run #8 of Test Case #1)

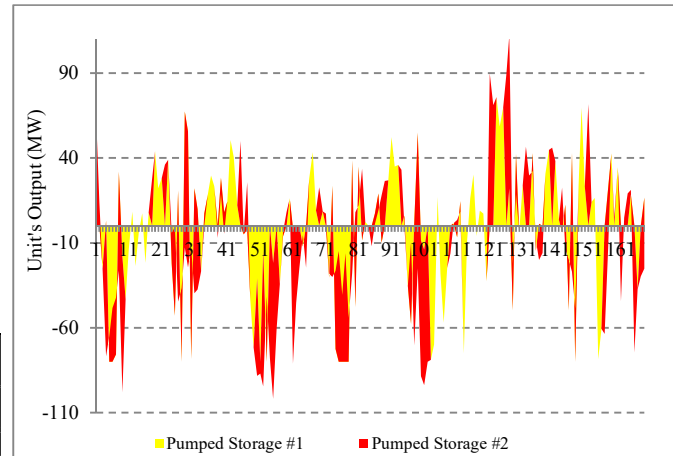


Figure 9. The Capacity of Pumped Storage Plants in Pumping and Generating Modes (Run #8 of Test Case #1)

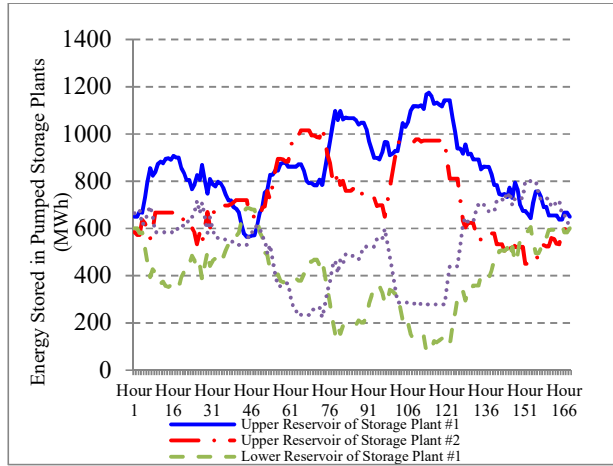


Figure 10. The Energy Stored in Upper and Lower Reservoirs of Pumped Storage Plants (Run #8 of Test Case #1)

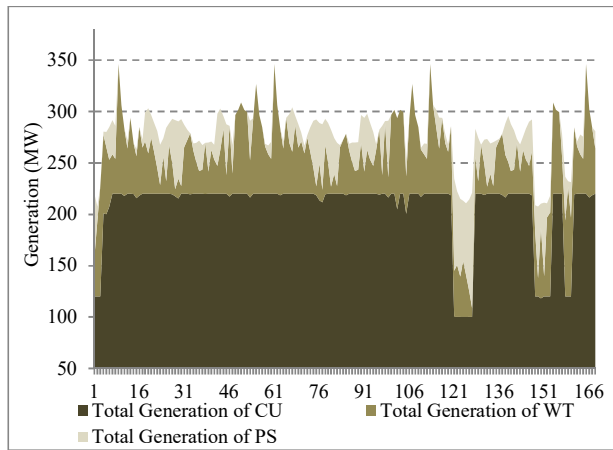


Figure 11. The Results of Generation by Conventional Units, Wind Farms and Pumped Storage Plants (Run #8 of Test Case #1)

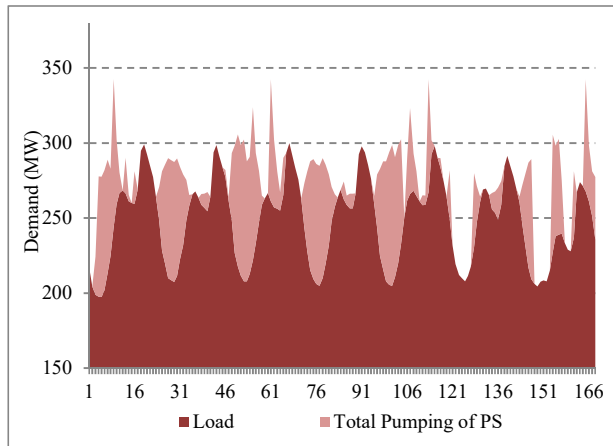


Figure 12. The Load and Demand of Pumping Mode of Pumped Storage Plants (Run #8 of Test Case #1)

### 5.2. Test system 2 (26C+2W+2PS)

The other test system has 26 conventional units (modified IEEE 24-bus system), two wind farms and two pumped storage plants that the data for these wind farms and pumped storage plants are given in previous section. The input data of conventional units of this test

system is given in [31] and [32], and also, the total peak load is 2700 MW. Other cost data for this test system is shown in [33].

Table 3 shows these results of UC problem using the proposed approach (based on type-2 MF) for the available wind power generation based on different value of aspiration level ( $J_0$ ). In this table, the value of objective function ( $z$ ) and MF value of other parameters have been shown. Based on criterion index, the Run #3 has the best result of proposed method implemented on UC problem.

Table 3: Results of 11 Runs of Type-2 Fuzzy Optimization Solution of Test Case #2 ( $\Delta W_{av} = \%5$  and

$$\Delta W'_{av} = \%2)$$

Run	$J_0$ (\$/week)	$z$	Total Profit (\$/week)	Criterion Index (CI)	Minimum Value of MF of Parameters in All Periods			
					$\mu_{P_p}$	$\mu_{P_R}$	$\mu_{w_1}(1)$	$\mu_{w_2}(2)$
#1	11,406,400	1	12,187,337	0.000	0.999	1	0.995	0.997
#2	13,406,400	0.952	13,112,962	0.488	0.951	0.953	0.975	0.950
#3	15,406,400	0.858	13,729,708	0.733	0.858	0.860	0.941	0.855
#4	17,406,400	0.746	13,718,887	0.633	0.746	0.750	0.894	0.897
#5	19,406,400	0.609	13,765,376	0.532	0.608	0.615	0.842	0.840
#6	21,406,400	0.594	13,878,894	0.556	0.594	0.600	0.834	0.834
#7	23,406,400	0.535	13,899,718	0.507	0.534	0.932	0.812	0.812
#8	25,406,400	0.485	13,918,600	0.465	0.484	0.768	0.791	0.792
#9	27,406,400	0.442	13,942,357	0.429	0.442	0.649	0.774	0.440
#10	29,406,400	0.405	13,963,077	0.398	0.405	0.511	0.757	0.402
#11	31,406,400	0.373	13,991,645	0.373	0.373	0.406	0.745	0.370

The UC results of each conventional unit supplying forecasted load during 168 hours (Run #3) is presented in Figure 13 and the pumped storage output in both pumping and generating modes is shown in Figure 14. The capacity of lower and upper reservoirs of each pumped storage plants are shown in Figure 15. Also, the results of total generation of all units (conventional units, wind farms and generating mode of pumped storage plants) and all demand (native load and pumping mode of pumped storage plants) are shown in Figure 16 and Figure 17 based on Run #3 of Table 3, respectively.

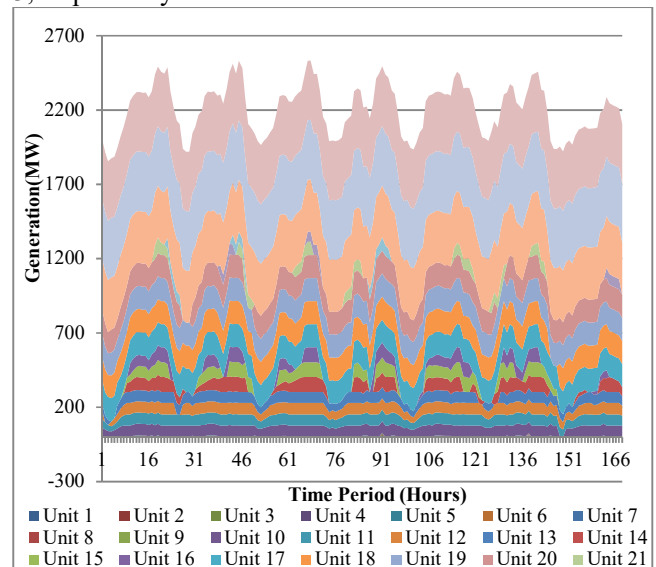


Figure 13. The UC results of Each Conventional Units Supplying Forecasted Load (Run #3 of Test Case #2)



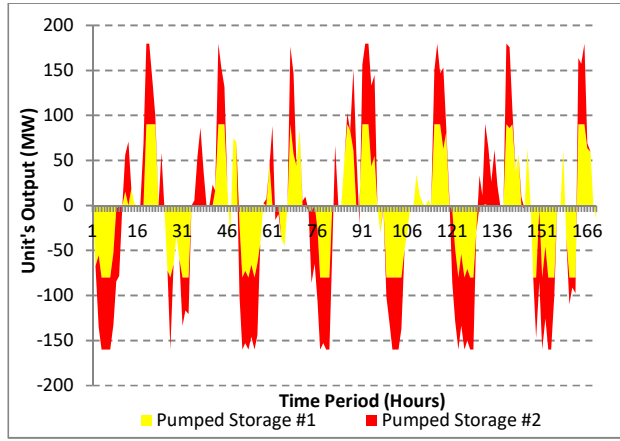


Figure 14. The Capacity of Pumped Storage Plants in Pumping and Generating Modes (Run #3 of Test Case #2)

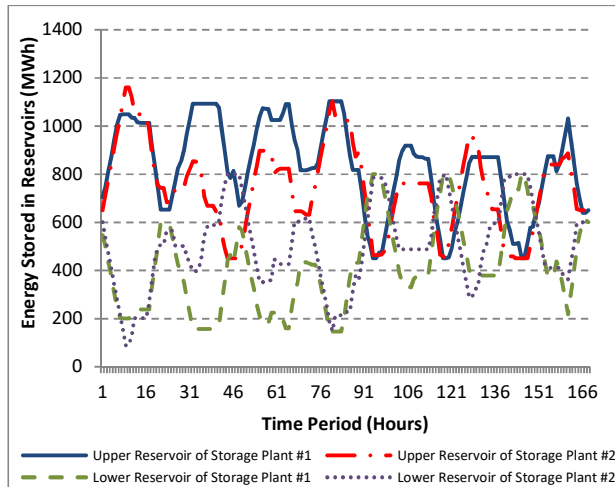


Figure 15. The Energy Stored in Upper and Lower Reservoirs of Pumped Storage Plants (Run #3 of Test Case #2)

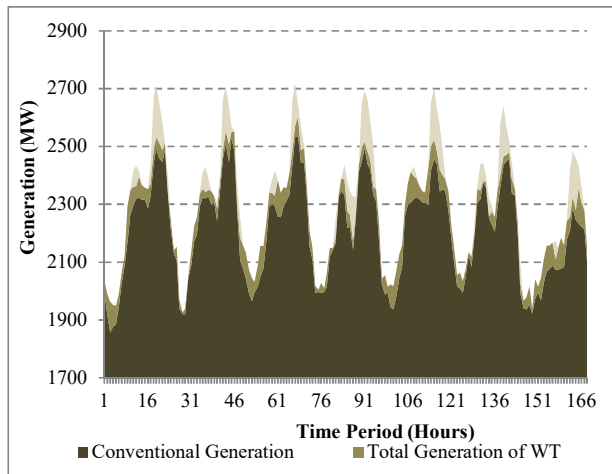


Figure 16. The Results of Generation by Conventional Units, Wind Farms and Pumped Storage Plants (Run #3 of Test Case #2)

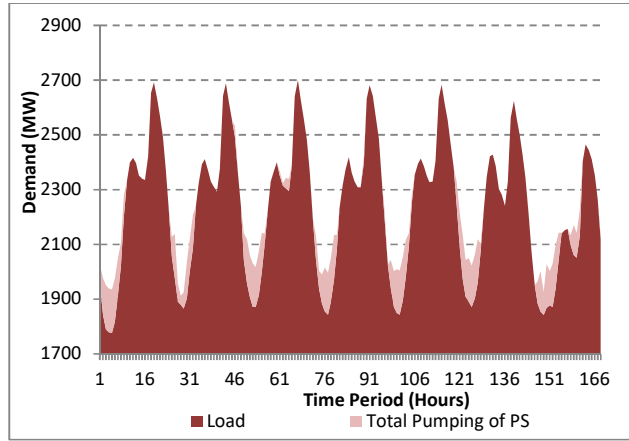


Figure 17. The Load and Demand of Pumping Mode of Pumped Storage Plants (Run #3 of Test Case #2)

Now, for the available wind power, the variation in the width of UMF when using type-2 fuzzy set ( $\Delta W_{av}$ ) is increased from 1% to 5% in steps of one percent, in each time period. The results of this sensitivity analysis have been presented in Table 4. The best objective function value is obtained in the case of 4% for upper and 1% for lower of type-2 fuzzy membership of wind power availability.

Table 4: Different FOU of Typ2-2 Fuzzy Optimization Solution of Test Case #2 (Based on Run #3 of Table 3)

$\Delta W_{av}$ (%)	$\Delta W'_{av}$ (%)	$Z$	Total Profit (\$/week)	Criterion Index (CI)	Minimum Value of MF of Parameters in All Periods			
					$\mu_{P_d}$	$\mu_{P_R}$	$\mu_{W_{av}}$	$\mu_{W'_{av}}$
5	5	0.8559	13,698,354	0.5242	0.8558	0.8751	0.8543	0.8538
5	4	0.8591	13,743,207	0.8007	0.8591	0.8641	0.8546	0.8575
5	3	0.8602	13,750,879	0.8486	0.8596	0.9174	0.8577	0.8582
5	2	0.6506	13,729,703	0.7171	0.8581	0.8605	0.8528	0.6395
5	1	0.8585	13,734,829	0.7494	0.8585	0.8605	0.8448	0.8417
4	4	0.8597	13,751,326	0.8513	0.8597	0.8619	0.8546	0.8575
4	3	0.8497	13,612,429	0.0000	0.8497	0.8547	0.8417	0.8462
4	2	0.8592	13,729,366	0.7151	0.8580	0.8603	0.8528	0.8542
4	1	0.8598	13,752,738	0.8600	0.8597	0.8619	0.8448	0.8417
3	3	0.8544	13,677,730	0.3975	0.8543	0.8566	0.8508	0.8524
3	2	0.8551	13,687,192	0.4556	0.8551	0.8580	0.8528	0.8513
3	1	0.8586	13,736,239	0.7580	0.8586	0.8630	0.8448	0.8417
2	2	0.8586	13,736,819	0.7615	0.8586	0.8618	0.8528	0.8542
2	1	0.8594	13,747,590	0.8275	0.8594	0.8784	0.8448	0.8417
1	1	0.8596	13,749,627	0.8409	0.8596	0.8791	0.8448	0.8417

## 6. SUMMARY AND CONCLUSIONS

A fuzzy optimization approach is presented for solving the unit commitment (UC) problem integrating large scale wind farms with pumped storage plants. This problem is firstly defined by a crisp optimization problem including uncertainty in some parameters, and then converted into a fuzzy formulation with a profit-based objective function. This UC problem was solved using the Mixed Integer Non Linear Programming (MINLP) method. In order to take into account the uncertainties in forecasted load, reserved power generation, and the available wind power, the type-1 and type-2 MFs are defined for these parameters. Numerical testing results clearly demonstrate the trade-off between maximizing total profit and satisfying the constraints. For a given desired profit, the fuzzy

optimization-based method can generate an optimal scheduling with its constraints' satisfaction. Therefore, this approach can provide information for generation scheduler to make the best trade-off between the profit (different desired profits) and constraints' satisfaction (different decision membership value  $z$ ).

This paper shows that the interval type-2 fuzzy set can be employed to efficiently model the linguistic uncertainty in the available wind power generation which exists in opinion of different experts. Different UC solutions are obtained using different MFs from different experts that led the problem in making decision for unit scheduling. The results of this paper demonstrated that the decision for unit commitment in an uncertain environment of type-1 fuzzy MF modeling can be obtained just by using a single type-2 fuzzy MF, when all type-1 MF are in the footprint of uncertainty (FOU) of type-2 MF.

### Nomenclature

$a_g, b_g, c_g$	The coefficients of generating unit $g$
$DT_g^r$	Minimum down time of unit $g$ , in number of time period
$El(s,t)$	Lower reservoir energy level of pumped storage $s$ at time period $t$ , in MWh
$El_{\max}(s)$	Lower reservoir energy capacity limit of pumped storage $s$ , in MWh
$EP(t)$	Forecasted energy price at time period $t$ , in \$/MWh
$Eu(s,t)$	Upper reservoir energy level of pumped storage $s$ at time period $t$ , in MWh
$Eu_{\max}(s)$	Upper reservoir energy capacity limit of pumped storage $s$ , in MWh
$g$	Index for thermal generating unit
$J$	Total profit of UC problem over all period
$M(s,t)$	Commitment state of pumped storage $s$ at time period $t$ (generation mode = 1, pumping mode = 0)
$N_G$	Number of thermal generating units
$N_S$	Number of pumped storage plants
$N_W$	Number of wind farms
$OMVCT(g)$	Operation and maintenance variable cost of thermal unit $g$ , in \$/MWh
$OMVCW(w)$	Operation and maintenance variable cost of wind farm $w$ , in \$/MWh
$P_d(t)$	System demand at time period $t$ , in MW
$P_{Gg,\min}$	Lower limit of thermal unit $g$ , in MW
$P_{Gg,\max}$	Upper limit of thermal unit $g$ , in MW
$P_{GD}(g,t)$	Load contribution of thermal unit $g$ at time period $t$ , in MW
$P_R(t)$	A fraction of total system load for system reserve requirement (first part) at time period $t$ , in MW

$P_{GR}(g,t)$	Reserve contribution of thermal unit $g$ at time period $t$ , in MW
$P_W(w,t)$	Generation of wind farm $w$ at time period $t$ , in MW
$P_{W,\max}$	Maximum generation of wind farm $w$ , in MW
$PS_{g,\max}(s)$	Maximum limit of generation mode of pumped storage $s$ , in MW
$PS_g(s,t)$	Generation mode of pumped storage $s$ at time period $t$ , in MW
$PS_p(s,t)$	Pumping mode of pumped storage $s$ at time period $t$ , in MW
$PS_{p,\max}(s)$	Maximum limit of pumping mode of pumped storage $s$ , in MW
$PSRES$	A fraction of pumped storage capacity contribute to operating reserve, in percent
$RD_g$	Maximum ramp down of unit $g$ , in MW per Hour
$RESW$	A fraction of total wind power employed to compensate wind power prediction errors, in percent
$RP(t)$	Forecasted reserve price at time period $t$ , in \$/MWh
$RU_g$	Maximum ramp up of unit $g$ , in MW per Hour
$s$	Index for pumped storage plant
$SU(g)$	Startup cost of thermal unit $g$ , in \$
$t$	Index for time period
$T$	Number of periods under study (168 Hours)
$t_g^{on}$	Duration time of unit $g$ is continuously on
$t_g^{off}$	Duration time of unit $g$ is continuously off
$TC$	Total operating costs
$TR$	Total revenues
$U(g,t)$	Commitment state of unit $g$ at time period $t$ (on = 1, off = 0)
$UT_g$	Minimum up time of unit $g$ , in number of time period
$V(w,t)$	Commitment state of wind farm $w$ at time period $t$ (on = 1, off = 0)
$w$	Index for wind farm
$W_{av}(w,t)$	Maximum available wind power of wind farm $w$ at time period $t$ , in MW
$WS(w,t)$	Wind speed in wind farm $w$ at time period $t$ , in m/sec
$\eta(s)$	Efficiency of pumping mode of pumped storage $s$

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